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The Role of Diversity and Network Coding in Random Access

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Knowledge for Tomorrow

Random Access: a modern "old" paradigm

- In spite of being possibly the simplest access schemes, random access protocols are still essential in most applications
- Slotted Aloha, introduced by Abramson in 1970 [1], has been adopted over years in many standards, typically for control channels
 - ▶ GSM, UMTS, DVB-RCS2, ...
 - often operated in the low-load / high-reliability region
- Recently, a new mass-market is drawing a renewed interest towards random access as a true data-delivery access scheme
 - ▶ M2M, Internet of Things, satellite AIS, sensor-data gathering, ...
 - myriad of (low-cost) uncoordinated devices with possibly sporadic traffic
 - quest for high-throughput and spectral efficiency

[1] N. Abramson, "The Aloha system - another alternative for computer communications," in Proc. of 1970 Fall Joint Computer Conf., 1970



A few relevant approaches devised to deal with collision



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MUD and Successive Interference Cancelation (SIC)

iteratively subtract interference contribution of decoded packets



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• iteratively subtract interference contribution of decoded packets

diversity (in time or frequency)

- increase chances of decoding by sending replicas
- Diversity Slotted Aloha (DSA) [2] protocol family

[2] G. Choudhury and S. Rappaport, "Diversity Aloha - a random access scheme for satellite communications," IEEE Trans. Commun., Mar. 1983.

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a combination of the two

- Contention Resolution DSA [3], Irregular Repetition Slotted Aloha [4]
- full efficiency for large MAC frames, very high efficiency in practical regions (0.8 pk/slot)

[2] G. Choudhury and S. Rappaport, "Diversity Aloha - a random access scheme for satellite communications," IEEE Trans. Commun., Mar. 1983.

- [3] E. Casini, R. D. Gaudenzi, and O. del Rio Herrero, "Contention resolution diversity slotted Aloha (CRDSA): An enhanced random access scheme for satellite access packet networks.," IEEE Trans.Wireless Commun., Apr. 2007.
- [4] G. Liva, "Graph-based analysis and optimization of contention resolution diversity slotted Aloha," IEEE Trans. Commun., Feb. 2011.



These approaches stem from a classical *single-hop, many-to-one* setting





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A different flavor is suggested from satellite topologies

• the presence of multiple receivers triggers spatial diversity w/o the need for additional transmissions



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- the presence of multiple receivers triggers spatial diversity w/o the need for additional transmissions
- viable setting for many applications



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 - WLAN, cellular networks, airborne networks, ...



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- two-layers: role of diversity and network coding



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- two-layers: role of diversity and network coding



2-layer many-to-many

Spatial diversity and Aloha

We focus on a relay-aided Slotted Aloha scheme, where *K* independent observations of a slot assumed to be available to enjoy *spatial diversity*

- relevant work on spatial (antenna) diversity under Rayleigh fading and capture effect by Zorzi in [5]
- we strive for a concise characterization that captures some relevant tradeoffs

Resorting to a simpler channel model, we achieve

- elegant and exact expressions of uplink throughput
- highlight key tradeoffs and design hints
- complement analysis with a downlink phase
 - dimensioning for arbitrary K based on Slepian-Wolf coding
 - ► A&F with SIC for 2 users case





[5] M. Zorzi, "Mobile Radio Slotted ALOHA with Capture, Diversity and Retransmission Control in the Presence of Shadowing," Wireless Networks, Aug 1998.

Outline

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- A characterization of the system uplink
 - the two-relay case
 - extension to a generic K
- Closing the loop: downlink strategies
 - an optimal solution based on RLNC
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- Conclusions



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System model - Uplink

- slotted system, one packet fits one slot
- infinite user population, channel accesses in one slot follow a Poisson distribution with intensity ρ
- on/off fading channel model [2]
 - independent packet erasure links with loss probability $\boldsymbol{\epsilon}$
 - if erased: no interference generated
- no MUD: either 1 pkt received, destructive collision, or nothing is seen
 - at each relay, slotted aloha with erasures

$$\mathcal{T}_{sa} = \sum_{u=0}^{\infty} \frac{\rho^u e^{-\rho}}{u!} u(1-\varepsilon)\varepsilon^{u-1} = \rho(1-\varepsilon)e^{-\rho(1-\varepsilon)}$$





[2] E. Perron, M. Rezaeian, and A. Grant, "The on-off fading channel," in Proc. IEEE Int. Symp. on Information Theory, Yokohama, Japan, Jul. 2003

Uplink system model

To capture the effect of diversity, we define a packet to be *collected* if it is received by at least one of the relays

Uplink performance evaluated in terms of

- packet loss probability ζ_K
 - probability of not being collected by the relay set
- uplink throughput: $\mathcal{T}_{up,K}$
 - average number of collected pkts per slot
 - captures key tradeoff of multiple receiver (diversity/duplicate packets)
 - amount of information that can be collected: upper bound for downlink strategies





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Uplink throughput : the two-relay case

General approach to throughput calculation

Let C be the number of collected packets over one slot, $C \in \{0, 1, ..., K\}$ $T_{up,K}$ evaluated by conditioning on the number u of users accessing the channel

$$\mathcal{T}_{up,K} = \mathbb{E}_{U}[\mathbb{E}[C | U]] = \sum_{u=0}^{\infty} \sum_{c=0}^{K} \frac{\rho^{u} e^{-\rho}}{u!} \cdot c \Pr\{C = c | U = u\}$$



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Two-relay case

- allows compact mathematical derivation
- practical relevance and easily implementable

$$\operatorname{Pr}\{C=1 \mid U=u\} = 2u(1-\varepsilon)\varepsilon^{u-1} \left[1-u(1-\varepsilon)\varepsilon^{u-1}\right] + u(1-\varepsilon)^2\varepsilon^{2(u-1)}$$

one relay receives, other does not due to erasures or a collision

both relays receive the same information unit

•
$$\Pr\{C=2 \mid U=u\} = u(u-1)(1-\varepsilon)^2 \varepsilon^{2(u-1)}$$



Uplink throughput: the two-relay case

$$\mathcal{T}_{up,2} = 2\rho(1-\varepsilon) e^{-\rho(1-\varepsilon)} - \rho(1-\varepsilon)^2 e^{-\rho(1-\varepsilon^2)}$$

twice throughput of SA loss due to having both relays decod

with erasures are information unit



peak throughput

- SA always 0.36
- diversity improves as $\boldsymbol{\epsilon}$ increases
- interest in evaluating maximum T_{up}
 - transcendental nature of T_{up,2} does not allow closed calculation
 - easy numerical solution

On the peak throughput with *K* = 2

- no erasures, no diversity gain
- larger ε favor de-correlation, improving throughput at the expense of reliability
- monotonically increasing trend, prior to plummeting to singularity for ϵ =1
- circled markers: system operated at ρ = 1/(1-ε), i.e., peak throughput of SA, very tight approximation

$$\mathcal{T}_{up,2}^*(\varepsilon) \simeq \frac{2}{e} - (1-\varepsilon) e^{-1-\varepsilon}, \quad 0 \le \varepsilon < 1$$



- efficiency close to 0.75 pk/slot with K=2, although in regions that are not practical
- > remarkable improvements in regions of interest, e.g., 15% w/ ϵ =0.1, 50% w/ ϵ =0.2
- no modification wrt SA needed to operate at maximum throughput



Uplink throughput: the general case

$$\mathcal{T}_{up,K} = \sum_{k=1}^{K} (-1)^{k-1} \binom{K}{k} \rho (1-\varepsilon)^k e^{-\rho(1-\varepsilon^k)}$$



Uplink throughput: the general case

Proposition: for an arbitrary number K of relays, the uplink throughput is given by

$$\mathcal{T}_{up,K} = \sum_{k=1}^{K} (-1)^{k-1} \binom{K}{k} \rho (1-\varepsilon)^k e^{-\rho(1-\varepsilon^k)}$$

Sketch of proof

by the weak law of large numbers

 $\mathcal{T}_{up,K} = \lim_{n \to \infty} \frac{|\bigcup_{k=1}^{K} \mathcal{A}_{k}^{n}|}{n} \qquad \text{set of pkts received by} \\ \text{relay } k \text{ over } n \text{ slots}$

• by inclusion exclusion principle

$$\left| \bigcup_{k=1}^{K} \mathcal{A}_{k}^{n} \right| = \sum_{\mathcal{S} \subseteq \{1, \dots, K\}, \mathcal{S} \neq \emptyset} (-1)^{|\mathcal{S}| - 1} \left| \mathcal{I}_{\mathcal{S}}^{n} \right|$$

$$\mathcal{I}^n_{\mathcal{S}} = \bigcap_{k \in \mathcal{S}} \mathcal{A}^n_k$$

cardinality only depends on |S|, so that

$$\left| \bigcup_{k=1}^{K} \mathcal{A}_{k}^{n} \right| = \sum_{k=1}^{K} (-1)^{k-1} \binom{K}{k} a_{k}^{n} \quad \text{for } k = |S|$$

- reception pattern over each slot is independent, over a single slot cardinality is either 0 or 1
- by weak law of large numbers,

$$\lim_{n \to \infty} \frac{a_k^n}{n} = \sum_{u=1}^{\infty} \frac{e^{-\rho} \rho^u}{u!} \binom{u}{1} \left((1-\varepsilon)\varepsilon^{u-1} \right)^{|\mathcal{S}|}$$

probability of having |S| nodes rx the same pkt

Uplink throughput – general case

$$\mathcal{T}_{up,K} = \sum_{k=1}^{K} (-1)^{k-1} \binom{K}{k} \rho (1-\varepsilon)^k e^{-\rho(1-\varepsilon^k)}$$

- diversity increases with number of relays
 - increased maximum number of collectable packets
 - reduced probability of common erasure/collision patterns

• $T_{up,K} > 1 \text{ pk/slot for } K \ge 4 \text{ (}\epsilon=0.2\text{)}$





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 - $T_{up,K} > 1 \text{ pk/slot for } K \ge 4 \text{ (}\epsilon=0.2\text{)}$
- peak throughput (numerical evaluation)
 - smaller gain by additional relays
 - Iogarithmic-like trend in K conjectured





Delivery reliability

- When uplink operated in peak throughput conditions, each user experiences a high loss probability (e.g., $1 e^{-1} \simeq 0.63$ with SA)
- Practical systems (e.g., logon and signaling channels) operated at much lower loads to enhance reliability

To evaluate impact of diversity on delivery reliability, we evaluate the probability ζ_{κ} that one transmitted packet is not retrieved by any relay

$$\zeta_K = \sum_{i=0}^{\infty} \Pr[\mathcal{O}]I = i] \Pr[\mathbf{I} = i]$$

outage event number of interferers

independent erasure patterns at each relay

$$\Pr[\mathcal{O}|I=i] = (1 - (1 - \varepsilon)\varepsilon^i)^K = \sum_{k=0}^K (-1)^k \binom{K}{k} \left((1 - \varepsilon)\varepsilon^i\right)^k$$



Delivery reliability

Averaging over the Poisson distribution of channel accesses, we get:

$$\zeta_K = \sum_{k=0}^{K} (-1)^k \binom{K}{k} (1-\varepsilon)^k e^{-\rho(1-\varepsilon^k)}$$

behavior increasing number of relays, in a low-load region

• ho
ightarrow 0: $\zeta_{\rm K}
ightarrow {\cal E}^{\rm K}$

- no erasures, maximum benefit of diversity
- practical operating regions, e.g., $\zeta_k = 5e-2$
 - not achievable w/ SA
 - 6- and 10-fold larger load supported when K=3 and K=4 wrt K=2





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The downlink problem

- uplink analysis characterized the reception capabilities of a set of relays subject to independent erasures
- complementary task: deliver the whole received information to a central gateway in an *uncoordinated* fashion, *employing the minimum number of resources*

Two main contributions

- provide an asymptotically optimal strategy based on RLNC
- characterize a DL scheme based on A&F and SIC for the special case K=2



Assumptions

- finite downlink bandwidth shared via TDMA
- reliable delivery of one pkt (possibly a linear combination of what received) over one DL slot



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Downlink rates

- sequences of packets received by relays over a time-span of n slots are correlated, as they stem from a common transmission pattern
- delivery to the gateway takes places in an uncoordinated fashion
 - > assignment of rates R_k , in terms of DL transmissions per each uplink slot
- distributed source coding problem (Slepian-Wolf)
 - presence of erasures can be overcome assuming that decoder knows position their Γ via packet headers

$$\sum_{k \in \mathcal{S}} R_k \ge H(W_{\mathcal{S}} | W_{\overline{\mathcal{S}}}, \Gamma), \quad \forall \ S \subseteq [1, 2, \dots, K]$$

• computation of entropies would require full joint probability distribution, so we resort to equivalent conditions:

$$\sum_{k \in \mathcal{S}} R_k \ge \mathcal{T}_{up,K} + \sum_{k=1}^{K-|\mathcal{S}|} (-1)^k \binom{K-|\mathcal{S}|}{k} \rho(1-\varepsilon)^k e^{-\rho(1-\varepsilon^k)}, \quad \forall \mathcal{S} \subseteq \{1,\dots,K\}$$



An optimal strategy based on RLNC

Proposition: A strategy based on Random Linear Network Coding achieves the lower bound on DL resources, so that the complete information content can be delivered to the GW with an overall rate equal to $T_{up,K}$

intuition for K = 2

transmission policy

- A: **transmit k**_A linear combinations
- B: transmit $\mathbf{k}_{\mathbf{B}} \cdot \mathbf{k}_{\mathbf{AB}} = n(T_{up} \cdot T_{sa})$ combinations

decoding at GW

- $k_{AB} = E[\# pkt in both queues inn slots] = n(2T_{sa}-T_{up})$
- k_{A}^{*} , k_{B}^{*} , k_{AB}^{*} = actual # of packets in the queues

$$\mathbf{H}\mathbf{x}^{\mathbf{T}} = \mathbf{c}^{\mathbf{T}} \qquad \begin{pmatrix} \mathbf{H}_{A} & \mathbf{H}_{AB} & \mathbf{0}_{A} \\ \mathbf{0}_{B} & \mathbf{H}_{BA} & \mathbf{H}_{B} \end{pmatrix} \begin{pmatrix} \mathbf{x}_{A}^{T} \\ \mathbf{x}_{AB}^{T} \\ \mathbf{x}_{B}^{T} \end{pmatrix} = \begin{pmatrix} \mathbf{c}_{A}^{T} \\ \mathbf{c}_{B}^{T} \end{pmatrix}$$

using A's combinations, retrieve \mathbf{x}_{A} and \mathbf{x}_{AB} : w.h.p. if $\mathbf{k}_{A} \ge \mathbf{k}_{A}^{*}$

using B's combinations and what retrieved, decode \mathbf{x}_{B} : w.h.p. if $k_{B}-k_{AB} \ge k_{B}^{*}-k_{AB}^{*}$

An alternative downlink approach

- The D&F downlink approach is simple and suited for uncoordinated relays, yet prevents joint decoding of uncorrelated signals at the GW
- Practical systems may offer larger bandwidth in the downlink phase (e.g., relays as satellites or as terrestrial base stations or APs)
 - the K=2 relays reliably forward on a slot-basis the analog waveform they received, even in the presence of a collision,
 - Successive Interference Cancellation (SIC) is applied at the GW



- for K=2, SIC helps if one relay experiences a collision involving the packet decoded at the other relay
- ideal SIC assumed, details on the accuracy in noisy channels in e.g., [4]



[4] G. Liva, "Graph-based analysis and optimization of contention resolution diversity slotted ALOHA," IEEE Trans. Comm., vol. 59, no. 2, Feb. 2011.

useful configurations

- the scheme enjoys a gain of one data unit over plain uplink whenever SIC can be applied:
 - conditioned on u, this has probability probability:

$$\frac{2u(u-1)\varepsilon^{u-1}(1-\varepsilon)}{\varepsilon^{u-2}(1-\varepsilon)^2}$$

x y relay A relay B



relay A decodes

collision w/ 2 packets at B









for an erasure rate of 0.2,
 66% and 20% gains of peak
 throughput wrt SA and uplink
 w/o SIC are achieved





• once more, optimal working load well approximated with the one of SA:

$$\mathcal{T}^*_{A\&F}(\varepsilon) \simeq \frac{2}{e} - e^{-1-\varepsilon} \left(1 - 3\varepsilon + 2\varepsilon^2\right)$$

- as opposed to reference uplink, there exist an optimal erasure rate, albeit not practical
- largest improvements (up to 25%) over non-SIC system experienced for erasure rates of interest



- simple SIC procedures wrt to those employed by practically implemented schemes, e.g., CRDSA
- interesting tradeoff between complexity and performance gain



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Some applications we look at for spatial diversity

piggybelly architecture [6]

- study on coverage provided by airliners equipped with transceivers over Europe's mainland
- most points on mainland are in visibility of several airliners simultaneously
- relevant for M2M and sensor-gathering, where ground transmitters perform random access and reception is improved by receiver-diversity
- multiple-satellite constellations
 - Constellations deployed for downlink services (e.g., Sirius Radio w/ 2 satellites in elliptical orbit)
 - possible enablers for return link with diversity (e.g., for mobile terminals in urban areas)

[6] S. Plass, M. Berioli, R. Hermenier, G. Liva, A. Munari, "M2M Architecture via Airliners", Transactions on Emerging Telecommunications Technologies, Jun. 2013.



Some applications we look at for spatial diversity

Some demonstration and prototyping activities ongoing at DLR

- implementation of low-complexity transmitters
 - TI CC2511F16 radio frontend controlled via Raspberry Pi
- ► SDR (ETTUS)-based receivers
- post-processing / interference cancellation and advanced signal processing techniques aided by a GPU-based implementation





Conclusions

- A slotted aloha system with multiple receivers has been analyzed
 - uplink phase: information collection at the receivers
 - downlink phase: information forwarded in an uncoordinated fashion to a central gateway
- under an on/off fading model, a complete characterization of the uplink in terms of throughput and delivery rate has been achieved
- bounds for the downlink transmission rates for complete information delivery have been derived
- a simple strategy based on network coding has been shown to achieve optimality
- an A&F downlink scheme with interference cancellation at the gateway has been studied for the two-relay case





Thank you!

