The Minimum List Size in List Decoding for Arbitrarily Varying Multiple Access Channel

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Background

(I)Ordinary Codes for Point to Point Arbitrarily Varying Channel (AVC)

- 1960 Blackwell-Breiman-Thomasian: introduced the model and determined capacity of random correlated codes $C_r(\mathcal{W})$
- 1978 Ahlswede: Capacities of random and deterministic codes may be different;
  elimination technique $\Rightarrow$
  Capacity of deterministic codes $\bar{C}(\mathcal{W}) = C_r(\mathcal{W})$ or 0
- 1985 Ericson: a sufficient condition (symmetrizable) for $\bar{C}(\mathcal{W}) = 0$
- 1988 Csiszar-Narayan: also necessary,
  to prove coding theorem without using elimination technique

Background

(II)List Decoding for Point to Point Arbitrarily Varying Channel (AVC)

- 1990 Pinsker conjectured, 1991 Ahlswede-C. proved: All $R < C_r(\mathcal{W})$ are achievable by deterministic list code of a constant list size(list size depending on $C_r(\mathcal{W}) - R$).
- The capacity of deterministic list codes $= 0$ or $C_r(\mathcal{W})$, as elimination technique works for list codes too. Thus to have the capacity of deterministic list codes, we only need to determine the the minimum list size $L_{\min}$ for which, deterministic list codes have a positive capacity.
  1995 Blinovsky-Narayan-Pinsker: determined $L_{\min}$ and showed *for binary AVC* $L_{\min} < \infty \Leftrightarrow C_r(\mathcal{W}) > 0$
- 1997 Hughes independently had $L_{\min}$ and showed *for all AVC* $L_{\min} < \infty \Leftrightarrow C_r(\mathcal{W}) > 0$

Background

(III) Ordinary Codes for Arbitrarily Varying Multiple Access Channel (AVMAC)

- 1981 Jahn: By an extension of elimination technique: capacity region $\bar{\mathcal{R}}(\mathcal{W})$ of deterministic codes has non-empty interior $\Rightarrow$ $\bar{\mathcal{R}}(\mathcal{W}) = \mathcal{R}_r(\mathcal{W})$ (capacity region of random correlated codes) and also determined $\mathcal{R}_r(\mathcal{W})$
  The conclusion is also true for list decoding, since elimination technique works for list codes as well

Next question: when capacity of deterministic codes has empty interior

- 1990 Gubner: "symmetrizable condition" is sufficient for capacity region to have empty interior and conjecture it is also necessary.
- 1999 Ahlswede-C.: proved the conjecture

Background

(IV) List Decoding for AVMAC
Since capacity region of deterministic list codes is equal to capacity region $\mathcal{R}_r(\mathcal{W})$ of random correlated codes in the case that it has a non-empty interior, naturally the next problem is to determine the minimum list size $L_{\min}$ for list codes to have a capacity region with a non-empty interior

- 2013 Nitinawarat: lower and upper bounds to the minimum list size $L_{\min}$ and showed
  for binary AVMAC $L_{\min} < \infty \Leftrightarrow \mathcal{R}_r(\mathcal{W})$ has a non-empty interior.
- a goal of this work: close the gap between the bounds

Coding for AVMAC

AVMAC: is defined by a set of MAC's

$$\mathcal{W} = \{W(\cdot|\cdot,\cdot,s) : \mathcal{X} \times \mathcal{Y} \to \mathcal{Z}, s \in \mathcal{S}\}.$$

When $\mathbf{x} \in \mathcal{X}^n, \mathbf{y} \in \mathcal{Y}^n$ are input to the channel, and state sequence $\mathbf{s} \in \mathcal{S}^n$ governs the channel, the channel outputs $\mathbf{z} \in \mathcal{Z}^n$ with probability

$$W^n(\mathbf{z}|\mathbf{x},\mathbf{y},\mathbf{s}) = \prod_{t=1}^{n} W(z_t|x_t, y_t, s_t)$$

Code: Codebooks $\mathcal{U} \subset \mathcal{X}^n, \mathcal{V} \subset \mathcal{Y}^n$ and decoding set $\mathcal{D}_{u,v}, u \in \mathcal{U}, v \in \mathcal{V}^n$
Ordinary code: $\mathcal{D}_{u,v} \cap \mathcal{D}_{u',v'} = \emptyset, (u,v) \neq (u',v')$
List code: $\mathcal{L}(\mathbf{z}) = \{(u,v) : \mathbf{z} \in \mathcal{D}_{u,v}\}$
List size of a list code: $L = \max_{\mathbf{z}} |\mathcal{L}(\mathbf{z})|$

Coding for AVMAC

The (average) probability of error for an (ordinary or list) code for an AVMAC is defined as

$$\max_{\mathbf{s}\in\mathcal{S}^n} \frac{1}{|\mathcal{U}|\mathcal{V}|} \sum_{u,v} W^n(\mathcal{D}^c_{u,v}|u,v),$$

where $\mathcal{D}^c_{u,v}$ is the compliment of $\mathcal{D}_{u,v}$.
Note: error criterions may make difference for capacity regions of AVMAC, but here we only consider the average probability of error.
A list code is called L-list code if its list size is no larger than L.
The capacity region of L-list codes for $\mathcal{W}$ is denoted by $\bar{\mathcal{R}}_L(\mathcal{W})$.
Problem: To find $L_{\min}(\mathcal{W})$, the minimum L such that the interior of $\bar{\mathcal{R}}_L(\mathcal{W})$ is not empty.

Coding for AVMAC

Remark: A random correlated code C_{K_1,K_2} for AVMAC is randomly distributed on a set $\{C_{k_1,k_2} : k_i \in \mathcal{K}_i, i = 1, 2\}$ of codes for the channel. Two random indices K_1 and K_2 are generated by two encoders, respectively. To perform a random code, the decoder must know the outputs of the random indices. By elimination technique, one can reduce the size of the indices $|\mathcal{K}_1| = |\mathcal{K}_2| = n^2$ such that the encoders may send the indices in a block with vanishing rates to the decoder, if the interior of the capacity region of deterministic (ordinary or list) codes is not empty. Consequently in this case, the capacity region of random correlated codes is achievable with deterministic codes.

By the technique, it is sufficient for us to have a code of "small rates" and so it greatly simplify the proofs of coding theorems.

Symmetrizable Conditions AVMAC

Coding for arbitrarily (point-to-point or multiple access) channel can be considered as 0-sum game with two players.
The first player: the communicator (sender and receiver), chooses coding scheme; and
The second player: the “jammer”, chooses a state sequence to disturb the communication.
Intuitively, “symmetrizable conditions” provide ways for jammer (randomly) to choose state sequences and win the game.

Symmetrizable Conditions AVMAC

Symmetrizable Conditions by Gubner, (for ordinary codes for AVMAC): There exits Q_0, Q_1, or Q_2, such that at least one of the following 3 equalities holds

$$\sum_{s} W(z|x,y,s)Q_0(s|x',y') = \sum_{s'} W(z|x',y',s')Q_0(s'|x,y),$$

for all x, x', y, y', z;
The decoder may not distinguish (x, y) and (x', y'), if the jammer randomly chooses a state according to Q_0 (for randomly chosen (x', y')).

$$\sum_{s} W(z|x,y,s)Q_1(s|x') = \sum_{s'} W(z|x',y,s')Q_1(s'|x),$$

for all x, x', y, z;
The decoder may not distinguish (x, y) and (x', y), if the jammer randomly chooses a state according to Q_1 (for randomly chosen x').

Symmetrizable Conditions AVMAC

$$\sum_{s} W(z|x,y,s)Q_2(s|,y') = \sum_{s'} W(z|x,y',s')Q_2(s'|y),$$

for all x, y, y', z.

The decoder may not distinguish (x, y) and (x, y'), if the jammer chooses a state according to Q_2 (for a randomly chosen y').

That is, the jammer will "win the game" if one of the three conditions holds, or in other words, $\mathcal{W}$ is symmetrizable.

The Results:

The capacity region of deterministic (ordinary) codes has non-empty interior, and therefore is equal to capacity region of random correlated codes (i.e., the communicator wins) if and only if the channel is non-symmetrizable (none of the three equalities holds).

Symmetrizable Conditions AVMAC

Differently, since list decoding alow overlaps of decoding sets, it is not sufficient for the jammer to win the game, because in the case that decoder may not distinguish two pairs of codewords, he can simply put both to the decoding list. That is, he needs a stronger condition.

Symmetrizable Conditions AVMAC

l-symmetrizable condition by Nitinawarat (for list codes for AVMAC):The jammer will win the game for the list size l.
Diagonal:There exists $Q^{(l)}$ such that

$$\sum_s W(z|x(1), y(1)), s) Q^{(l)}(s|x(2), \ldots, x(l+1), y(2), \ldots, y(l+1))$$
$$= \sum_s W(z|x(\pi(1)), y(\pi(1)), s)$$
$$Q^{(l)}(s|x(\pi(2)), \ldots, x(\pi(l+1)), y_(\pi(2)), \ldots, y(\pi(l+1))),$$

for all $x(i) \in \mathcal{X}, i = 1, 2, \ldots l+1, y(j) \in \mathcal{Y}, j = 1, 2, \ldots, l+1$, $z \in \mathcal{Z}$, permutation π on $[l+1]$.
The decoder may not distinguish $(x(k), y(k)), k = 1, 2, \ldots, l+1$, if the jammer randomly chooses a state according to $Q^{(l)}$.

Symmetrizable Conditions AVMAC

l-symmetrizable condition by Nitinawarat (for list codes for AVMAC) (continue)

Rectangle: or exists $Q^{(a\times b)}$ with $(a+1)(b+1)\geq l+1$ such that

$$\sum_s W(z|x(1),y(1),s)Q^{(a\times b)}(s|x(2),\dots,x(a+1),y(2),\dots,y(b+1)$$

$$=\sum_s W(z|x(\pi(1)),y(\sigma(1)),s)$$

$$Q^{(a\times b)}(s|x(\pi(2)),\dots,x(\pi(a+1)),y(\sigma(2)),\dots,y(\sigma(b+1))),$$

for all
$x(i)\in\mathcal{X}, i=1,2,\dots a+1, y(j)\in\mathcal{Y}, j=1,2,\dots,b+1, z\in\mathcal{Z}$
and permutation π on $[a+1]$ and permutation σ on $[b+1]$.
The decoder may not distinguish $(x(i),y(j))$, $i=1,2,\dots,a+1$, $j=1,2\dots b+1$, if the jammer randomly chooses a state, according to $Q^{(a\times b)}$.

Symmetrizable Conditions AVMAC

l-symmetrizable condition by Nitinawarat (for list codes for AVMAC) (continue)
Let $L_{\min}(\mathcal{W})$ be the minimum L such that $\bar{\mathcal{R}}_L(\mathcal{W})$ a non-empty interior and $l_{\max}(\mathcal{W})$ be the maximum l such that $\mathcal{W}$ is l-symmetrizable. Then Nitinawarat proved:

$$l_{\max}(\mathcal{W}) + 1 \leq L_{\min}(\mathcal{W}) \leq (l_{\max}(\mathcal{W}) + 1)^2. \tag{1}$$

The gap is large if $l_{\max}(\mathcal{W})$ is large.
Why did he fail to have a tight bound?

- The possible configurations of the lists are too complicated
- The l-symmetrizable condition is too simple.

Symmetrizable Conditions AVMAC

Intuitively an equality in a symmetrizable condition implies that to have a small probability of error, a type of configurations must appear in lists. For example, 3-symmetrizable condition says the following configurations with 4 pairs of codeowords must appear in lists, and so the jammer wins the game for the list size 3:

- 3-Diagonal:
  $\{(\mathbf{x}(1), \mathbf{y}(1)), (\mathbf{x}(2), \mathbf{y}(2)), (\mathbf{x}(3), \mathbf{y}(3)), (\mathbf{x}(4), \mathbf{y}(4))\}$
- 3-Rectangle:
  $\{(\mathbf{x}(1), \mathbf{y}(1)), (\mathbf{x}(1), \mathbf{y}(2)), (\mathbf{x}(2), \mathbf{y}(1)), (\mathbf{x}(2), \mathbf{y}(2))\}$,
  $\{(\mathbf{x}(1), \mathbf{y}(1)), (\mathbf{x}(2), \mathbf{y}(1)), (\mathbf{x}(3), \mathbf{y}(1)), (\mathbf{x}(4), \mathbf{y}(1))\}$,
  $\{(\mathbf{x}(1), \mathbf{y}(1)), (\mathbf{x}(1), \mathbf{y}(2)), (\mathbf{x}(1), \mathbf{y}(3)), (\mathbf{x}(1), \mathbf{y}(4))\}$,
- But much more possible configurations for lists of 4 pairs of codewords are not included by 3-symmetrizable condition, e.g,
  $\{(\mathbf{x}(1), \mathbf{y}(1)), (\mathbf{x}(1), \mathbf{y}(2)), (\mathbf{x}(1), \mathbf{y}(3)), (\mathbf{x}(2), \mathbf{y}(2))\}$,
  $\{(\mathbf{x}(1), \mathbf{y}(1)), (\mathbf{x}(2), \mathbf{y}(1)), (\mathbf{x}(2), \mathbf{y}(2)), (\mathbf{x}(3), \mathbf{y}(2))\}$,
  ......

configurations of the lists

We introduce bipartite graphs (without isolated vertex) $B = (\{\mathcal{I}, \mathcal{J}\}, \mathcal{E})$ to describe configurations of the lists.
B is called a configuration of the list $\mathcal{L}(\mathbf{z})$ if there exist *bijections*

$$f : \mathcal{I} \to \{u \in \mathcal{U} : \exists v \in \mathcal{V} \text{ with } (u, v) \in \mathcal{L}(\mathbf{z})\}$$

and

$$g : \mathcal{J} \to \{v \in \mathcal{V} : \exists u \in \mathcal{U} \text{ with } (u, v) \in \mathcal{L}(\mathbf{z})\}$$

such that

$$\mathcal{L}(\mathbf{z}) = \{(f(i), g(j)) : (i, j) \in \mathcal{E}\}.$$

B' is contained by $\mathcal{L}(\mathbf{z})$ as a configuration, if it is a subgraph of a configuration B of $\mathcal{L}(\mathbf{z})$.

configurations of the lists

Informally, the bipartite graph is contained by a list if it can be embed into the list.

A bipartite graph is contained by a list code, as a configuration, if it contained by any decoding list of the code.

Let $\mathcal{B}$ be set of bipartite graphs. For $\mathcal{B}' \subset \mathcal{B}$, we call a list code a $\mathcal{B}'$-list code, if its all configurations are in $\mathcal{B}'$. The capacity region of $\mathcal{B}'$-list codes for AVMAC $\mathcal{W}$ is denoted by $\bar{\mathcal{R}}_{\mathcal{B}'}(\mathcal{W})$.

"$\mathcal{B}'$-list codes" contains "all information" of "L-list code", but not vice versa.

Obviously, a $\mathcal{B}'$-list codes is a L-list code, if no bipartite graph in $\mathcal{B}'$ has more than L edges.

B-Symmetrizable Condition AVMAC

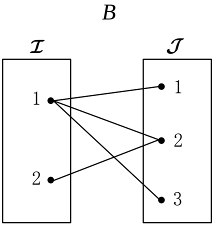
For a bipartite graph $B = (\{\mathcal{I}, \mathcal{J}\}, \mathcal{E})$, $\mathcal{W}$ is B-symmetrizable, if there exist a set of stochastic matrices $Q_{B,(i,j)}, (i,j) \in \mathcal{E}$ such that for all $|\mathcal{I}|$-tuple $x(\mathcal{I}) = (x(i), i \in \mathcal{I})$ labeled by $i \in \mathcal{I}$ and $|\mathcal{J}|$-tuple $y(\mathcal{J}) = (y(j), j \in \mathcal{J})$ labeled by $j \in \mathcal{J}$, $(i,j), (i',j') \in \mathcal{E}$ and z,

$$\sum_s W(z|x(i), y(i), s) Q_{B,(i,j)}(s|x(\mathcal{I} \setminus \{i\}), y(\mathcal{J} \setminus \{j\}))$$
$$= \sum_s W(z|x(i'), y(i'), s) Q_{B,(i',j')}(s|x(\mathcal{I} \setminus \{i'\}), y(\mathcal{J} \setminus \{j'\})),$$

where $x(\mathcal{I} \setminus \{i\}) = (x_i, i \in \mathcal{I} \setminus \{i\})$ and $y(\mathcal{J} \setminus \{j\}) = (y_j, j \in \mathcal{J} \setminus \{j\})$.
The decoder does not know for which “edge” (i,j), $(x(i), y(j))$ is the correct input pair, if the jammer chooses a state, according to $Q_{B,(i',j')}$, for a randomly chosen “edge” (i',j').
Denote the set of bipartite graphs symmetrizing $\mathcal{W}$, by

$$\mathcal{B}_0(\mathcal{W}) = \{B : \mathcal{W} \text{ is } B\text{-symmetrizable.}\}$$

Examples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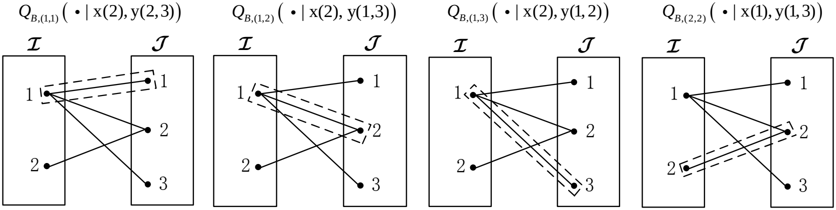
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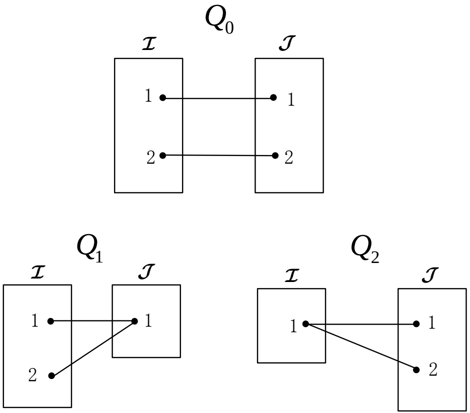
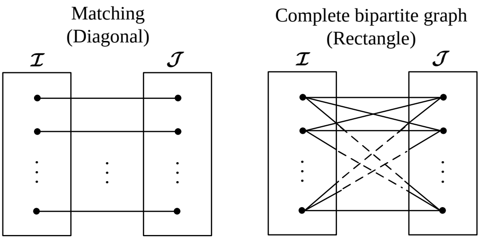
Examples

Gubner's symetrizable condition contains
all bipartite graphs with 2 edges



Example

But Nitnawarat's symetrizable condition only contains 2 types of bipartite graphs



The Main Results

An AVMAC $\mathcal{W}$ is finite symmetrizable if the cardinality of the set $|\mathcal{B}_0(\mathcal{W})| < \infty$.

Theorem 1

For a finite symmetrizable AVMAC, a list code with positive rates and probability of error smaller than a positive constant (depending on the channel), contains all members in $\mathcal{B}_0(\mathcal{W})$ as configurations, if the length of code is sufficiently large.

Theorem 2

Given a finite symmetrizable AVMAC, for any $\lambda > 0$ and sufficiently large n, there exists a $\mathcal{B}_0(\mathcal{W})$-list code of length n with probability of error smaller than λ, and positive rates. That is, $\bar{\mathcal{R}}_{\mathcal{B}_0(\mathcal{W})}(\mathcal{W})$ has a non-empty interior.

The Main Results

Certainly, we want a list code to contain as few as possible configurations of lists, under the condition that the error probability is arbitrarily small. Then the two theorems together say that $\mathcal{B}_0(\mathcal{W})$ is the set of configurations contained by the "best codes".
Theorems 1 and 2 have the following direct consequences.

Corollary 1

If $\mathcal{W}$ is finite symmetrizable and $|\mathcal{B}'| < \infty$,

$$\bar{\mathcal{R}}_{\mathcal{B}'}(\mathcal{W}) \begin{cases} = \mathcal{R}^*(\mathcal{W}) & \text{if } \mathcal{B}_0(\mathcal{W}) \subset \mathcal{B}' \\ \text{has an empty interior} & \text{else.} \end{cases}$$

Corollary 2

$$L_{\min}(\mathcal{W}) = \Lambda_{\max}(\mathcal{W}),$$

where $\Lambda_{\max}(\mathcal{W})$ is the maximum sizes of edge sets of bipartite graphs in $\mathcal{B}_0(\mathcal{W})$.

The Main Results

Now the remained questions for list decoding for AVMAC only are:

- When is an AVMAC finite symmetrizable?
- What can we say about list decoding for a non-finite symmetrizable AVMAC?

Let us first answer the first question:

Theorem 3

An AVMAC is finite symmetrizable, if and only if its capacity region of random correlated codes has a non-empty interior.

The Main Results

As capacity regions of all constant list codes are contained by capacity region of random correlated codes (Fano). The answer to the second question immediately follows from Theorem 3.

Corollary 3

For no finite L, the capacity region $\bar{\mathcal{R}}_L(\mathcal{W})$ has non-empty interior, if $\mathcal{W}$ is not finite symmetrizable. That is, $L_{\min}(\mathcal{W}) < \infty$ if and only if the capacity region of random correlated codes has a non-empty interior.

Outline of Proof of Theorem 1

To prove Theorem 1, we need to show, if a sufficiently long code does not contain a $B \in \mathcal{B}_0(\mathcal{W})$ as a configuration, then there exists a state sequence for which the error probability is larger than a constant. In other words, we only need to find a random strategy for the jammer to win the game in this case.

- The jammer first randomly and uniformly chooses a edge $(\tilde{I}, \tilde{J})$ in the bipartite graph B.
- Then he randomly, independently, and uniformly chooses $|\mathcal{I}| - 1$-tuple $\mathbf{X}(\mathcal{I} \setminus \{\tilde{I}\})$ of codewords and $|\mathcal{J}| - 1$-tuple $\mathbf{Y}(\mathcal{J} \setminus \{\tilde{J}\})$ of codewords from the two codebooks respectively, (for given $(\tilde{I}, \tilde{J}) = (i, j)$) as the "rest vertices" in B.
- Finally, for given $(\tilde{I}, \tilde{J}) = (i, j)$, $\mathbf{X}(\mathcal{I} \setminus \{i\}) = \mathbf{x}(\mathcal{I} \setminus \{i\})$ and $\mathbf{Y}(\mathcal{J} \setminus \{j\}) = \mathbf{y}(\mathcal{J} \setminus \{j\})$, the jammer randomly generates a sequence $\mathbf{S}$ be from $\mathcal{S}^n$ acccording to conditional distribution $Q^n_{B,(i,j)}(\cdot|\mathbf{x}(\mathcal{I} \setminus \{i\}), \mathbf{Y}(\mathcal{J} \setminus \{j\}))$.

Outline of Proof of Theorem 1

- In a view of game theory, the criterion of average probability of error means the jammer knows the coding scheme but not the codewords to be sent. As we consider average probability of error, we let words X and Y sent by the senders (communicator) be independently uniformly distributed on two codebooks and independent of the random strategy of the jammer ($(\tilde{I}, \tilde{J})$, $\mathbf{X}(\mathcal{I} \setminus \{\tilde{I}\})$, $\mathbf{Y}(\mathcal{J} \setminus \{\tilde{J}\})$ and S).
- Then by regrouping the terms in the expectation of error probability in X, Y, S, we show the the expectation is larger than $\lambda_0 := \frac{1}{4\Lambda_{\max}(\mathcal{W})}$ (a constant).
- Consequently, there exists a state sequence for which the error probability is larger than λ_0.

Outline of Proof of Theorem 2

We need to prove that for any $\mathcal{W}$ and $\lambda > 0$, there exists a list code containing no $B \notin \mathcal{B}_0(\mathcal{W})$ as a configuration.

- *codebook:* The two codebooks are randomly, and independently generated from two sets of typical sequences $\mathcal{T}_X^n$ and $\mathcal{T}_Y^n$ with $\min_x P_X(x), \min_y P_Y(y) \geq a$ for a fixed small $a > 0$, respectively. By Chernov bound, we show the codebooks have some "*nice properties*".

Outline of Proof of Theorem 2

- *Decoding:* Let $\bar{I}$ ($\bar{J}$) be the maximum size of the first (second) parts of vertex sets of the bipartite graphs in $\mathcal{B}_0(\mathcal{W})$. An output sequence $\mathbf{z}$ is decoded as (u, v), if and only if there is an $\mathbf{s} \in \mathcal{S}^n$ and a quadruple (X, Y, S, Z) of random variables with $(\mathbf{x}(u), \mathbf{y}(v), \mathbf{s}, \mathbf{z}) \in \mathcal{T}^n_{XYSZ}$ satisfying simultaneously the below conditions.

  O)
  $$D(P_{XYSZ} \| P_X \times P_Y \times P_S \times W) < \xi, \tag{2}$$
  for a $W \in \mathcal{W}$.

  I) For all $\bar{I}$-tuple of codewords $\mathbf{x}^{\bar{I}}$ in the first codebook, and $\bar{J}$-tuple $\mathbf{y}^{\bar{J}}$ in the second codebook having joint type $P_{XYSZX^{\bar{I}}Y^{\bar{J}}}$ with (X, Y, S, Z),
  $$I(XYZ; X^{\bar{I}}Y^{\bar{J}}|S) < \zeta. \tag{3}$$

Outline of Proof of Theorem 2

- Next, we show the decoding rules O) and I) guarantee that there is no $B \notin \mathcal{B}_0(\mathcal{W})$ contained by the code as a configuration, if ξ *and* ζ *are chosen sufficiently small* (depending only on $\mathcal{W}$).
- It is shown that by the "nice properties" of the randomly generated codebooks, the error probability is exponentially vanishing as length of the code increases, if *the rates of the code are "much smaller" than* ξ *and* ζ.

Note the rates of the code may be very small, but it is sufficient for us to show the theorem, because of the elimination technique.

Outline of Proof of Theorem 3

Recall the capacity region $\mathcal{R}^*(\mathcal{W})$ of random correlated codes is the closure of the convex hull of sets

$$\{(R_1, R_2) : 0 \leq R_1 \leq \inf_{\bar{W} \in \bar{\mathcal{W}}} I(X; Z|Y),$$
$$0 \leq R_2 \leq \inf_{\bar{W} \in \bar{\mathcal{W}}} I(Y; Z|X),$$
$$0 \leq R_1 + R_2 \leq \inf_{\bar{W} \in \bar{\mathcal{W}}} I(XY; Z)\}$$

for all random inputs X, Y, and output Z, where

$$\bar{\mathcal{W}} = \{\sum_s Q(s) W(\cdot|\cdot, \cdot, s), \text{ for all distribution } Q \text{ on } \mathcal{S}\}.$$

Outline of Proof of Theorem 3

(1) Then obviously $\mathcal{R}^*(\mathcal{W})$ has a non-empty interior if and only if
(p1)

$$\max_{P_X, P_Y} \inf_{\bar{W} \in \bar{\mathcal{W}}} I(X; Z|Y) > 0;$$

(p2)

$$\max_{P_X, P_Y} \inf_{\bar{W} \in \bar{\mathcal{W}}} I(Y; Z|X) > 0,$$

simultaneously hold, which imply that
(p0)

$$\max_{P_X, P_Y} \inf_{\bar{W} \in \bar{\mathcal{W}}} I(XY; Z) > 0.$$

That is, $\mathcal{R}^*(\mathcal{W})$ has a non-empty interior if and only if (p0), (p1), and (p2) simultaneously hold.

Outline of Proof of Theorem 3

(2) Next, we prove that an AVMAC $\mathcal{W}$ is finite symmetrizable if and only if the following conditions simultaneously hold.
(P1) There is a upper bound $D_1 > 0$ such that no bipartite graph $B = (\{\mathcal{I}, \mathcal{J}\}, \mathcal{E}) \in \mathcal{B}_0(\mathcal{W})$ having a vertex in $\mathcal{J}$ with degree lager than D_1;
(P2) There is an upper bound $D_2 > 0$ such that no bipartite graph $B = (\{\mathcal{I}, \mathcal{J}\}, \mathcal{E}) \in \mathcal{B}_0(\mathcal{W})$ having a vertex in $\mathcal{I}$ with degree lager than D_2;
(P0) There is an upper bound $D_0 > 0$, such that no bipartite graph in $\mathcal{B}_0(\mathcal{W})$ having a matching larger than D_0.
(3) Finally we show (Pk) is equivalent to (pk), respectively, for k=0,1,2. This completes our proof.

A Challenging Problem

To avoid a heavy calculation, in all proofs of direct coding theorems for AVMAC, were based on the elimination technique that is, to construct a code with very small positive rates, instead to achieve the capacity regions.

But, elimination technique does NOT work for channels with state constraint (i.e., the jammer has to pay cost (or power) for the state chosen and he can only pay the limited cost).

Can we find a coding scheme directly to achieve the capacity regions (even for ordinary deterministic codes for AVMAV), without using elimination technique?

Thank You!