Channel Capacity in the Non-asymptotic Regime: Taylor-type Expansion, Computable Benchmarks and Applications to LTE

En-Hui Yang

University of Waterloo, Canada

October 26, 2012
Joint work with Jin Meng

Outline

Outline

1. Introduction
2. Taylor-type Expansion
3. Computable Benchmark
4. Application to LTE

Channel Capacity in the Non-Asymptotic Regime

- Consider a discrete input memoryless channel (DIMC) $P = \{p(y|x) : x \in \mathcal{X}, y \in \mathcal{Y}\}$ with arbitrary output alphabet $\mathcal{Y}$.
- By channel capacity, we often meant in the literature Shannon capacity

$$C = \max_X I(X;Y).$$

  In this talk, however, the notion of channel capacity will be used abusively to also denote non-asymptotic counterparts of C, in particular, the following quantities:

Non-asymptotic Channel Capacity with Type Constraint

$R_{t,n}(\epsilon)$: the best channel coding rate achievable with block length n and codeword type t subject to error probability ϵ.

Non-asymptotic Channel Capacity without Type Constraint

$$R_n(\epsilon) = \max_t R_{t,n}(\epsilon)$$

- Shannon Capacity [Shannon]

$$\lim_{n\to+\infty} R_n(\epsilon) = C \text{ for any } 0 < \epsilon < 1$$

- Second Order Analysis [Strassen; Polyanskiy, Verdu and Poor; Hayashi]

$$\lim_{n\to+\infty} \frac{\sqrt{n}(C - R_n(\epsilon))}{\sigma_D(P)} = Q^{-1}(\epsilon) \text{ for a constant } \epsilon$$

- Moderate Deviation Analysis [Altug and Wagner; Polyanskiy and Verdu]

$$\lim_{n\to+\infty} \frac{-\ln \epsilon_n}{n\rho_n^2} = \frac{1}{2\sigma_D^2(P)}$$

for $R_n(\epsilon_n) = C - \rho_n$ and $o(1) \geq \rho_n \geq \omega\left(\frac{1}{\sqrt{n}}\right)$.

- Error Exponent [Fano; Gallager; and many others]

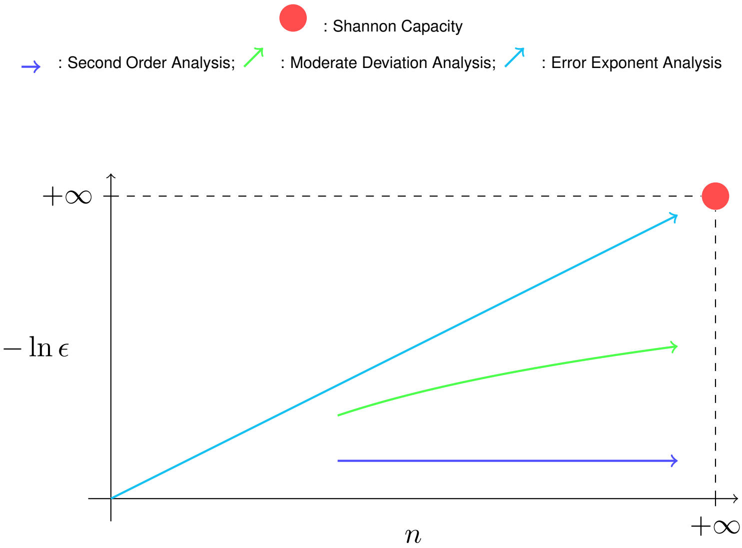
$$\lim_{n\to+\infty} \frac{-\ln \epsilon_n}{n} = E_r(R)$$

for $R_c \leq R_n(\epsilon_n) = R < C$.

Tools in Asymptotic Analysis

Asymptotic Analysis	Tool
Shannon Capacity	AEP, Law of Large Number
Error Exponent	Large Deviation Theory
Moderate Deviation Analysis	Moderate Deviation Theory
Second Order Analysis	Berry-Esseen Central Limit Theorem

Graphical Interpretation of Asymptotic Analysis

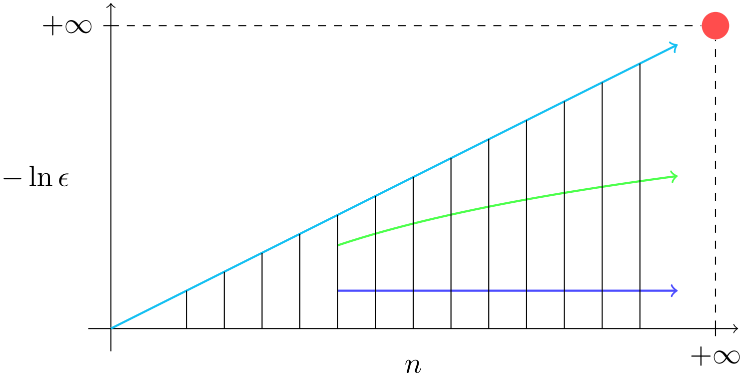


Graphical Interpretation of Asymptotic Analysis

● : Shannon Capacity

→ : Second Order Analysis; ↗ : Moderate Deviation Analysis; ↗ : Error Exponent Analysis

| : Non-asymptotic Regime



Non-asymptotic Analysis: Generic Bounds

- Error Exponent Bounds [Fano; Gallager; and many others]
- Generic one-shot bounds [Polyanskiy-Poor-Verdu Bounds and many others listed in their IT 2010 paper]
- For example, Fano's 1961 achievability bound for a discrete memoryless channel and Shannon random code ensemble

$$L(x^n, y^n) = \ln \frac{p(y^n|x^n)}{q(y^n)} \text{ and } q(y^n) = \prod_{i=1}^{n} q(y_i)$$

$$\begin{aligned}
&\Pr\{\text{error}|c_0\} \\
&\quad \leq \Pr\{\text{error}, L(c_0, Y^n) \leq L_0\} + \Pr\{\text{error}, L(c_0, Y^n) > L_0\} \\
&\quad \leq \Pr\{L(c_0, Y^n) \leq L_0\} \\
&\qquad + \Pr\{\exists m \neq 0, L(c_0, Y^n) > L_0, L(c_m, Y^n) \geq L(c_0, Y^n)\}
\end{aligned}$$

The major effort was on how to bound the second probability.

- Many subsequent achievability bounds followed more or less the same approach by spending tremendous effort on the second probability based on ML or Feinstein's threshold decoding.

Advantage

- Generic one-shot bounds look neat and are applicable to any channel.
- They are generally very tight if one can find an effective way to compute them, which has happened so far only for special channels such as BSC and BEC.

Disadvantage

- Error exponent bounds are generally not tight in the non-asymptotic regime when the coding rate is very close to the Shannon capacity
- The evaluation of generic one-shot bounds is very challenging for general channels, especially those without certain symmetric properties.
- Generic one-shot achievability bounds may not be applicable to codes with structures.

Outline

Key Definitions

Let P be a channel with transition probability $p(y|x)$, discrete input alphabet $\mathcal{X}$ and discrete or continuous output alphabet $\mathcal{Y}$, and t be a type of sequences in $\mathcal{X}^n$.

$$
\begin{aligned}
q_t(y) &\triangleq \sum_{x\in\mathcal{X}} t(x)p(y|x) \\
q_t(y^n) &\triangleq \prod_{i=1}^{n} q_t(y_i) \\
I(t;P) &\triangleq \sum_{x\in\mathcal{X}} t(x)\int p(y|x)\ln\frac{p(y|x)}{q_t(y)}dy \\
P_{t,\delta} &\triangleq \Pr\left\{\ln\frac{p(Y^n|X^n)}{q_t(Y^n)} \le I(t;P)-\delta \,\middle|\, X^n = x^n\right\}
\end{aligned}
$$

Jar Decoding

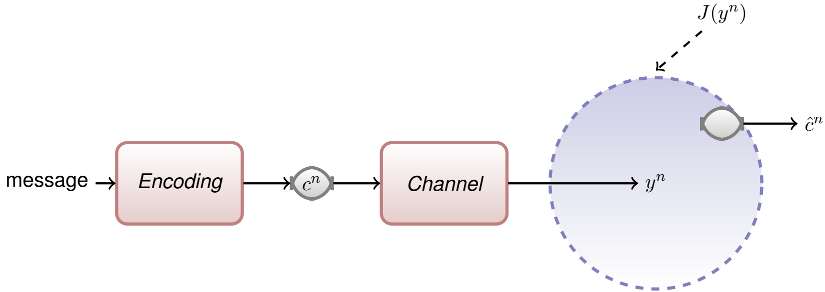


Figure: Jar Decoding

DIMC Jar

$$J(y^n) = \left\{ x^n \in \mathcal{T}_n^t : \frac{1}{n} \ln \frac{p(y^n|x^n)}{q_t(y^n)} > I(t;P) - \delta \right\}$$

Key Idea of Achievability Proof

Error probability is broken into two parts.

$$
\begin{aligned}
P_e &\leq \Pr\{\text{error}, c^n \notin J(Y^n)\} + \Pr\{\text{error}, c^n \in J(Y^n)\} \\
&\leq \Pr\{c^n \notin J(Y^n)\} + \Pr\{\exists \hat{c}^n \neq c^n, \hat{c}^n \in J(Y^n)\} \\
&\leq P_{t,\delta} + \sum_{x^n \in J(y^n)/\{c^n\}} \Pr\{x^n \text{ is a codeword}\}
\end{aligned}
$$

When the channel coding rate is close to Shannon Capacity,

- $P_{t,\delta}$ is the dominating term in P_e.
- the second part can be simply tackled by bounding the size of jar.

Non-asymptotic Achievability via Jar Decoding

Let $\mathcal{C}_{n,k}^{(1)}$ and $\mathcal{C}_{n,k}^{(2)}$ denote Elias' Generator and Gallager's parity check ensembles respectively.

Theorem (Achievability on Random Linear Code Ensemble)

For any binary input memoryless channel P and $i = 1, 2$,

$$P_e(\mathcal{C}_{n,k}^{(i)}) \leq \epsilon$$

whenever $R(\mathcal{C}_{n,k}^{(i)}) \leq \ln 2 - H(X|Y) - \delta + \frac{\ln(\epsilon - P_\delta^{(i)})}{n}$, and

$$P_\delta^{(i)} = C^{(i)} \Pr\left\{-\frac{1}{n}\ln p(X^n|Y^n) > H(X|Y) + \delta\right\} < \epsilon \qquad (1)$$

where X^n is a binary uniform i.i.d sequence, Y^n is the response of the channel P to X^n and $C^{(i)} = \begin{cases} 1 & i = 1 \\ \frac{1}{1-2^{-n}} & i = 2 \end{cases}$.

Let $\mathcal{C}_{t,n}$ denote Shannon random code ensemble with type t constraint and block length n.

Theorem (Achievability on Shannon Random Code Ensemble with Type Constraint)

For any discrete input memoryless channel P, any $t \in \mathcal{P}_n$ and δ,

$$P_e(\mathcal{C}_{t,n}) \leq \epsilon$$

whenever

$$R(\mathcal{C}_{t,n}) \leq I(t;P) - \delta + \frac{\ln(\epsilon - P_{t,\delta})}{n} - \frac{|\mathcal{X}|\ln(n+1)}{n} \quad (2)$$

and

$$P_{t,\delta} < \epsilon. \quad (3)$$

Theorem (Converse)

For any channel code $\mathcal{C}_{t,n}$ of block length n and codeword type t with rate $R(\mathcal{C}_{t,n})$ and average word error probability $P_e(\mathcal{C}_{t,n}) = \epsilon$,

$$R(\mathcal{C}_{t,n}) \leq I(t;P) - \delta - \frac{\ln(P_{t,\delta} - \epsilon)}{n} + \frac{\ln P(B_{t,n,\delta})}{n} \quad (4)$$

for any δ, as long as

$$\epsilon < P_{t,\delta}. \quad (5)$$

NEP with respect to relative entropy

$$\underline{\xi}_{D,-}(t; P, \lambda, n) e^{-nr_-(t,\delta)} \leq P_{t,\delta} \leq \bar{\xi}_{D,-}(t; P, \lambda, n) e^{-nr_-(t,\delta)}$$

where $\lambda = \frac{\partial r_-(t,\delta)}{\partial \delta}$.

$$r_-(t,\delta) \triangleq \sup_{\lambda \geq 0} \left[\lambda(\delta - I(t; P)) - \sum_{x \in \mathcal{X}} t(x) \ln \int p(y|x) \left[\frac{p(y|x)}{q_t(y)} \right]^{-\lambda} dy \right]$$

An Example of $r_-(t, \delta)$

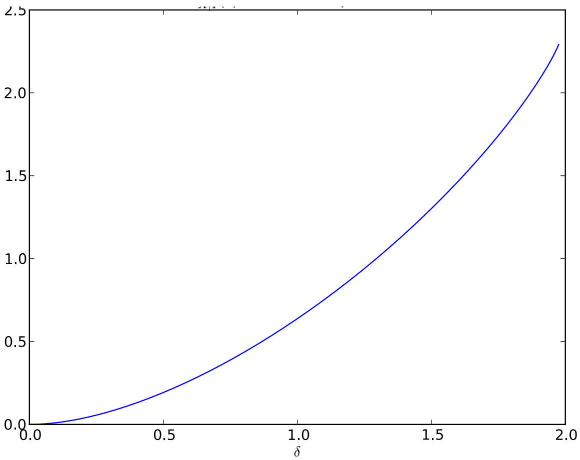


Figure: $r_-(t, \delta)$ for BSC with uniform t

Let X_t and $Y_{t,\lambda}$ be random variables with joint distribution $t(x)p(y|x)f_{-\lambda}(y|x)$ where

$$f_{-\lambda}(y|x) \triangleq \frac{\left[\frac{p(y|x)}{q_t(y)}\right]^{-\lambda}}{\int p(v|x)\left[\frac{p(v|x)}{q_t(v)}\right]^{-\lambda} dv}. \tag{6}$$

$$\sigma^2_{D,-}(t;P,\lambda) \triangleq \mathbf{E}\left\{\mathbf{Var}\left[\ln \frac{p(Y_{t,\lambda}|X_t)}{q_t(Y_{t,\lambda})}\middle| X_t\right]\right\} \tag{7}$$

$$M_{D,-}(t;P,\lambda) \triangleq \mathbf{E}\left\{\mathbf{M_3}\left[\ln \frac{p(Y_{t,\lambda}|X_t)}{q_t(Y_{t,\lambda})}\middle| X_t\right]\right\} \tag{8}$$

$$\hat{M}_{D,-}(t;P,\lambda) \triangleq \mathbf{E}\left\{\hat{\mathbf{M}}_\mathbf{3}\left[\ln \frac{p(Y_{t,\lambda}|X_t)}{q_t(Y_{t,\lambda})}\middle| X_t\right]\right\} \tag{9}$$

$$\begin{aligned}\bar{\xi}_{D,-}(t;P,\lambda,n) &\triangleq \frac{2C_{BE}M_{\mathrm{D},-}(t;P,\lambda)}{\sqrt{n}\sigma^3_{\mathrm{D},-}(t;P,\lambda)} \\ &+ e^{\frac{n\lambda^2\sigma^2_{D,-}(t;P,\lambda)}{2}}Q(\sqrt{n}\lambda\sigma_{D,-}(t;P,\lambda)) \\ &- e^{\frac{n\lambda^2\sigma^2_{D,-}(t;P,\lambda)}{2}}Q(\rho^* + \sqrt{n}\lambda\sigma_{D,-}(t;P,\lambda))\end{aligned} \tag{10}$$

$$\underline{\xi}_{D,-}(t;P,\lambda,n) \triangleq e^{\frac{n\lambda^2\sigma^2_{D,-}(t;P,\lambda)}{2}}Q(\rho_* + \sqrt{n}\lambda\sigma_{D,-}(t;P,\lambda)) \tag{11}$$

where $Q(\rho^*) = \frac{C_{BE}M_{\mathrm{D},-}(t;P,\lambda)}{\sqrt{n}\sigma^3_{\mathrm{D},-}(t;P,\lambda)}$, $Q(\rho_*) = \frac{1}{2} - \frac{2C_{BE}M_{\mathrm{D},-}(t;P,\lambda)}{\sqrt{n}\sigma^3_{\mathrm{D},-}(t;P,\lambda)}$, and C_{BE} is the universal constant in the Berry-Esseen central limit theorem.

Definition of $\delta_{t,n}(\epsilon)$

The solution of δ to the equation

$$e^{\frac{n\lambda^2\sigma_D^2(t;P,\lambda)}{2}} Q(\sqrt{n}\lambda\sigma_D(t;P,\lambda))e^{-nr_-(t,\delta)} = \epsilon$$

where $\lambda = \frac{\partial r_-(t,\delta)}{\partial \delta}$.

Interpretation of $\delta_{t,n}(\epsilon)$

- $P_{t,\delta} \approx \epsilon$ when $\delta = \delta_{t,n}(\epsilon)$.
- Major term of the rate penalty from $I(t;P)$ due to n and ϵ.
- Relative magnitude of n and ϵ.

Taylor Expansion of $R_{t,n}(\epsilon)$

Expansion of $R_{t,n}(\epsilon)$ with respect to $\delta_{t,n}(\epsilon)$

Under some mild conditions, it is shown

$$|R_{t,n}(\epsilon) - [I(t;P) - \delta_{t,n}(\epsilon)]| \leq o(\delta_{t,n}(\epsilon))$$

whenever $\epsilon < \frac{1}{2}$, where

$$o(\delta_{t,n}(\epsilon)) = r_{-}(t, \delta_{t,n}(\epsilon)) + \frac{(|\mathcal{X}| + 1)\ln(n+1) + d}{n}.$$

Taylor Expansion of $R_n(\epsilon)$

Expansion of $R_n(\epsilon)$ with respect to $\delta_{t,n}(\epsilon)$

Under some mild conditions, it is shown

$$|R_n(\epsilon) - [I(t^*; P) - \delta_{t^*,n}(\epsilon)]| \leq o(\delta_{t^*,n}(\epsilon))$$

whenever $\epsilon < \frac{1}{2}$, where t^* is defined as

$$t^* = \arg\min_t [I(t; P) - \delta_{t,n}(\epsilon)]$$

and

$$o(\delta_{t^*,n}(\epsilon)) = r_-(t^*, \delta_{t^*,n}(\epsilon)) + \frac{(|\mathcal{X}| + 1.5)\ln(n+1) + d}{n}.$$

Consistency

When ϵ is constant or non-exponentially decreasing with respect to n and $n \to +\infty$,

- $\delta_{t,n}(\epsilon) \to 0$;
- t^* is always within a small neighbourhood of the capacity-achieving input distribution;
- $C - \frac{\sigma_D(P)}{\sqrt{n}} Q^{-1}(\epsilon)$ is very close to $I(t^*; P) - \delta_{t^*,n}(\epsilon)$;
- and therefore, the result is consistent with the second order analysis and moderate deviation analysis.

Comparison with Asymptotic Analysis

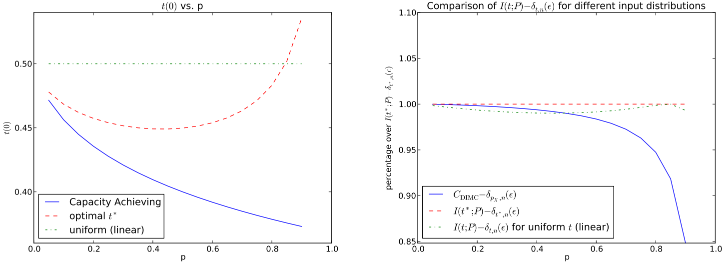
Divergence

When n is finite and ϵ is relatively small with respect to n,

- $\delta_{t,n}(\epsilon)$ is not relatively small;
- t^* is not necessarily capacity-achieving;
- even when t^* is capacity-achieving (in some symmetric cases) but the second term in the following expansion is not small,

$$r_-(t,\delta) = \frac{1}{2\sigma_D^2(t;P)}\delta^2 + \frac{-\hat{M}_D(t;P)}{6\sigma_D^6(t;P)}\delta^3 + O(\delta^4)$$

  $\delta_{t^*,n}(\epsilon)$ does not agree with $\frac{\sigma_D(P)}{\sqrt{n}}Q^{-1}(\epsilon)$;
- and therefore, the taylor expansion can yield more reliable approximation of $R_n(\epsilon)$.



$t(0)$ vs. p

$\frac{I(t;P)-\delta_{t,n}(\epsilon)}{I(t^*;P)-\delta_{t^*,n}(\epsilon)}$ for different t

Figure: Illustration for the Z channel with $n = 1000$ and $\epsilon = 10^{-6}$: (a) comparison of t^* with the capacity achieving distribution; and (b) comparison of $I(t;P) - \delta_{t,n}(\epsilon)$ among different distributions t.

Outline

Approximations

Second-Order Approximation

$$R_n^{SO}(\epsilon) = I(t^*; P) - \delta_{t^*,n}(\epsilon)$$

NEP Approximation

$$R_n^{NEP}(\epsilon) = I(t^*; P) - \delta_{t^*,n}(\epsilon) - \frac{\ln \epsilon}{n} + \frac{\ln P(B_{t^*,n,\delta_{t^*,n}(\epsilon)})}{n}$$

Normal Approximation

$$R_n^{Normal}(\epsilon) = C - \frac{\sigma_D(P)}{\sqrt{n}} Q^{-1}(\epsilon)$$

Second-Order Approximation

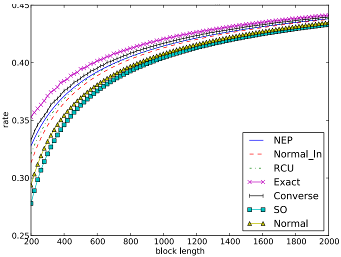
$$R_n^{SO}(\epsilon) = I(t^*; P) - \delta_{t^*,n}(\epsilon)$$

NEP Approximation

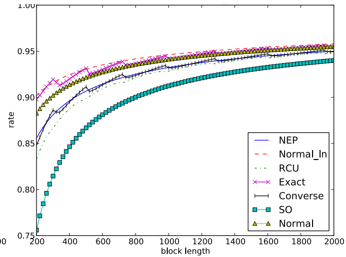
$$R_n^{NEP}(\epsilon) = I(t^*; P) - \delta_{t^*,n}(\epsilon) - \frac{\ln \epsilon}{n} + \frac{\ln P(B_{t^*,n,\delta_{t^*,n}(\epsilon)})}{n}$$

Normal Approximation

$$R_n^{Normal}(\epsilon) = C - \frac{\sigma_D(P)}{\sqrt{n}} Q^{-1}(\epsilon) + \frac{\ln n}{2n}$$



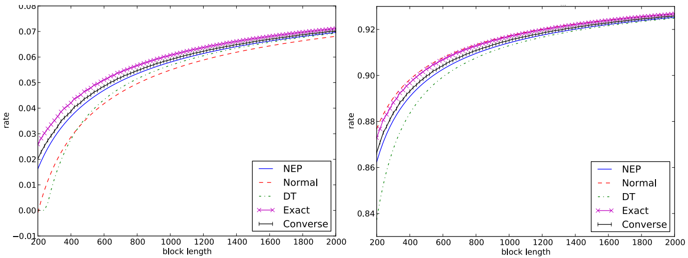
$p = 0.11$ and $\epsilon = 10^{-3}$



$p = 0.001$ and $\epsilon = 10^{-6}$

Figure: BSC with different cross-over probability p and error probability ϵ

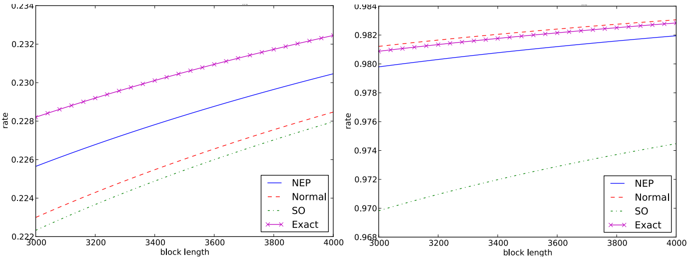
Numerical Comparison of Approximations - BEC



$p = 0.9$ and $\epsilon = 10^{-6}$ $\quad$ $p = 0.05$ and $\epsilon = 10^{-6}$

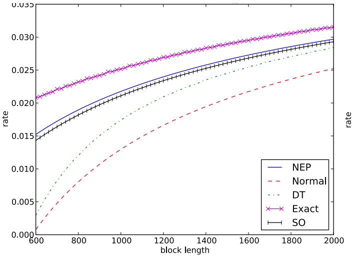
Figure: BEC with different erasure probability p and error probability ϵ

Numerical Comparison of Approximations - BIAGC

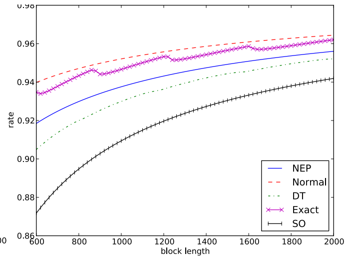


SNR$= -3.52$dB and $\epsilon = 10^{-3}$ SNR$= 9.63$dB and $\epsilon = 10^{-9}$

Figure: BIAGC with different SNR and error probability ϵ



$p = 0.9$ and $\epsilon = 10^{-6}$



$p = 0.001$ and $\epsilon = 10^{-9}$

Figure: Z Channel with different $p = \Pr\{Y = 1|X = 0\}$ and error probability ϵ

1

[1] Changing t^* to the capacity-achieving input distribution will lower the bounds DT and Exact.

Outline

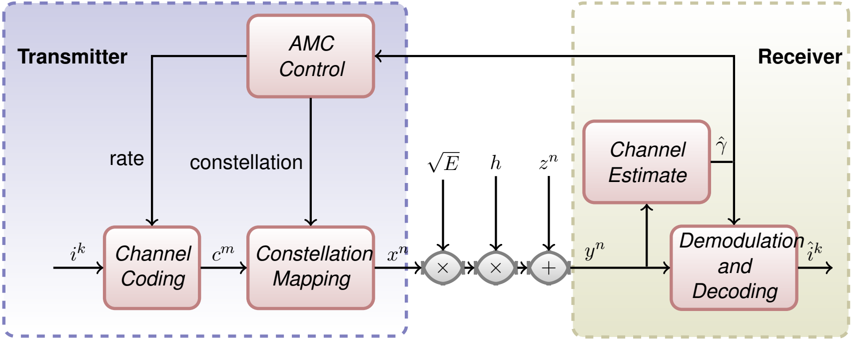


Figure: Adaptive Modulation and Coding System

$$\frac{1}{n}\sum_{i=1}^{n}|x_i|^2 = 1. \; Z \sim \mathcal{CN}\left(0, \frac{\sigma^2}{2}\right). \; \gamma = \frac{h^2 E}{\sigma^2}.$$

Constellation and Coding Rate Selection

Definition

- Effective Coding Rate: $r = \frac{k}{m}$.
- Spectral Efficiency: $R = \frac{k}{n}$.
- Constellation $\mathcal{X}$: $R = r \log_2 |\mathcal{X}|$.
- Error Probability: $\Pr\{\hat{i}^k \neq i^k\}$.
- System Throughput: $th = (1 - \epsilon)R$.

Given several constellations and a channel code with adjustable coding rate, how to select constellation and coding rate to match channel condition (or simply snr) in order to maximize the system throughput?

Optimal Achievable Spectral Efficiency and Throughput of Modulation and Coding over AWGN Channel

Let $\mathcal{X}$ denote the modulation constellation and t denote the type of the codeword of modulation and coding scheme. A constellation is said to be normalized with respect to t if

$$\sum_{x \in \mathcal{X}} t(x)|x|^2 = 1.$$

Given a type t and a normalized constellation $\mathcal{X}$, let $P_{\mathcal{X},\gamma}$ denote the AWGN channel with discrete input $\mathcal{X}$ and snr γ.

Optimal Achievable Spectral Efficiency and Throughput

$R_{\mathcal{X},t,n}(\gamma, \epsilon)$ and $th_{\mathcal{X},t,n}(\epsilon)$ are optimal achievable spectral efficiency and throughput respectively with block length n and modulation codeword type t subject to error probability ϵ over AWGN channel $P_{\mathcal{X},\gamma}$.

Approximation of $R_{\mathcal{X},t,n}(\gamma, \epsilon)$ and $th_{\mathcal{X},t,n}(\epsilon)$ based on Taylor Expansion

Approximation of $R_{\mathcal{X},t,n}(\gamma, \epsilon)$

$$R_{\mathcal{X},t,n}(\gamma, \epsilon) \approx I(t; P_{\mathcal{X},t}) - \delta_{t;P_{\mathcal{X},\gamma},n}(\epsilon)$$

Approximation of $th_{\mathcal{X},t,n}(\epsilon)$

$$th_{\mathcal{X},t,n}(\epsilon) \approx \max_{\epsilon}(1-\epsilon) R_{\mathcal{X},t,n}(\gamma, \epsilon)$$

Denote the optimal solution ϵ of above maximization problem by $\epsilon^{th}_{\mathcal{X},t,n}(\gamma)$ and $R_{\mathcal{X},t,n}(\gamma, \epsilon^{th}_{\mathcal{X},t,n}(\gamma))$ by $R^{th}_{\mathcal{X},t,n}(\gamma)$.

Here $\delta_{t,n}(\epsilon)$ is redefined as $\delta_{t;P_{\mathcal{X},\gamma},n}(\epsilon)$ to emphasize its dependency on the channel.

Suppose $\{\mathcal{X}_i\}_{i=1}^{m}$ are available constellations in an adaptive modulation and coding system.

Selecting Rule

S1 For any snr γ, calculate $th_{\mathcal{X}_i,t,n}(\gamma)$ for $1 \le i \le m$ and determine

$$i^* = \arg\max_{1 \le i \le m} th_{\mathcal{X}_i,t,n}(\gamma).$$

S2 Select $\mathcal{X}_{i^*}$ as the desired constellation, and calculate the desired ECR according to

$$r = \frac{R^{th}_{\mathcal{X}_{i^*},t,n}(\gamma)}{\log_2 |\mathcal{X}_{i^*}|}.$$

Considering the snr penalty incurred by the practical implementation of demodulation and decoding, the selecting rule is modified as follows.

Modified Selecting Rule

S1 Determine, for each constellation $\mathcal{X}_i$, the the snr penalty $\Delta\gamma_i$.

S2 Calculate $th_{\mathcal{X}_i,t,n}(\gamma - \Delta\gamma_i)$ for $1 \leq i \leq m$ and determine

$$i^* = \arg\max_{1\leq i\leq m} th_{\mathcal{X}_i,t,n}(\gamma - \Delta\gamma_i). \tag{12}$$

S3 Select $\mathcal{X}_{i^*}$ as the desired constellation, and calculate the desired ECR according to

$$r = \frac{R^{th}_{\mathcal{X}_{i^*},t,n}(\gamma - \Delta\gamma_i)}{\log_2 |\mathcal{X}_{i^*}|}. \tag{13}$$

Adaptive Modulation and Coding in LTE System

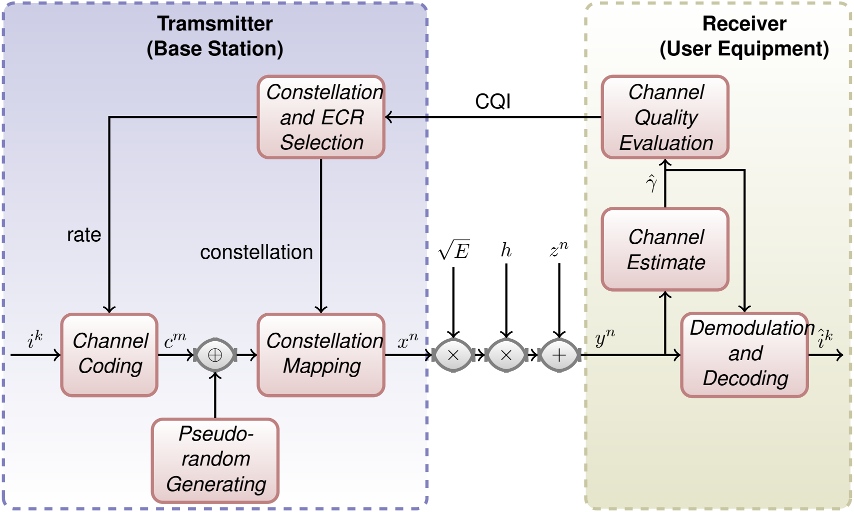


Figure: Adaptive Modulation and Coding in the LTE system

CQI Table

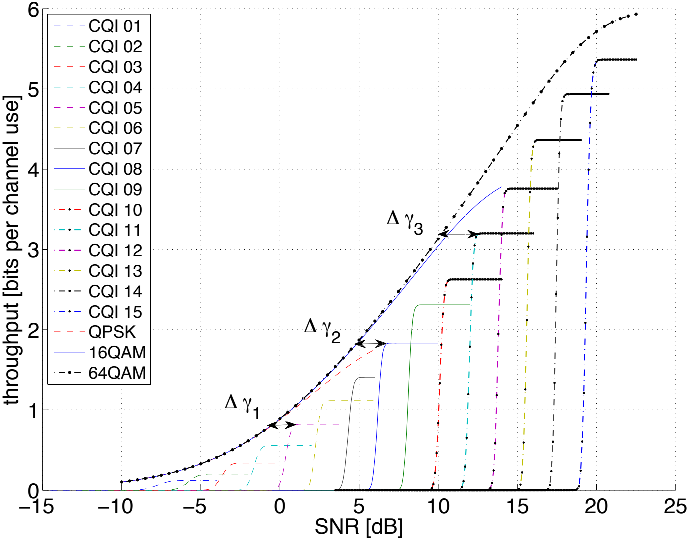
CQI Index	Modulation	ECR	MPR
1	QPSK	78/1024	0.1523
2	QPSK	120/1024	0.2344
3	QPSK	193/1024	0.3770
4	QPSK	308/1024	0.6016
5	QPSK	449/1024	0.8770
6	QPSK	602/1024	1.1758
7	16QAM	378/1024	1.4766
8	16QAM	490/1024	1.9141
9	16QAM	616/1024	2.4063
10	64QAM	466/1024	2.7305
11	64QAM	567/1024	3.3223
12	64QAM	666/1024	3.9023
13	64QAM	772/1024	4.5234
14	64QAM	873/1024	5.1152
15	64QAM	948/1024	5.5547

In LTE system, the selection of constellation and coding rate based on channel snr is made at the receiver side, considering the fact that there may exist different implementation of demodulation and decoding at the receiver side.

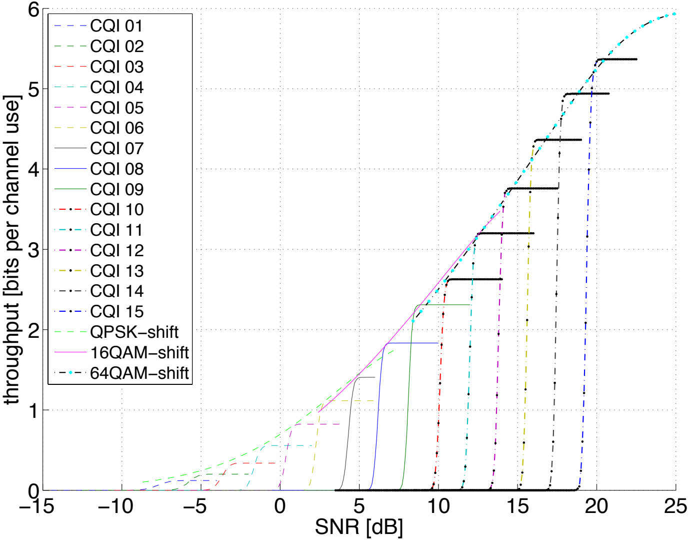
CQI Reporting Rule

The receiver should send the highest CQI index under the combination of constellation and coding rate of which the error probability $\leq 10^{-1}$ can be achieved.

Determine $\Delta\gamma$ for QPSK, 16QAM and 64QAM.

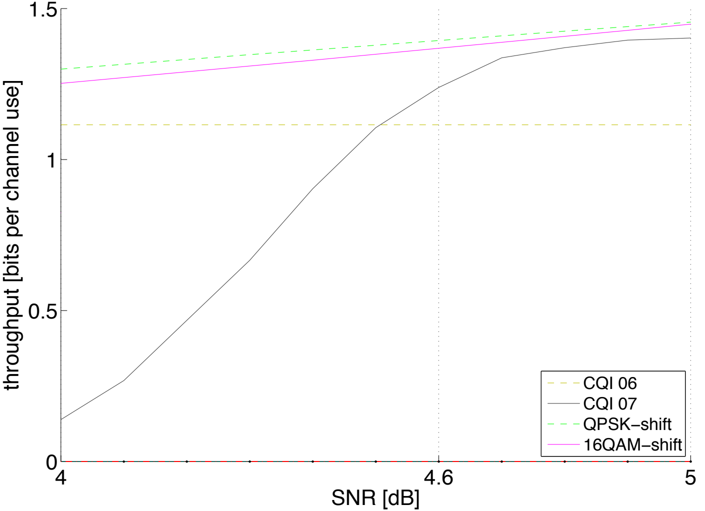


Apply Modified Selecting Rule to LTE System



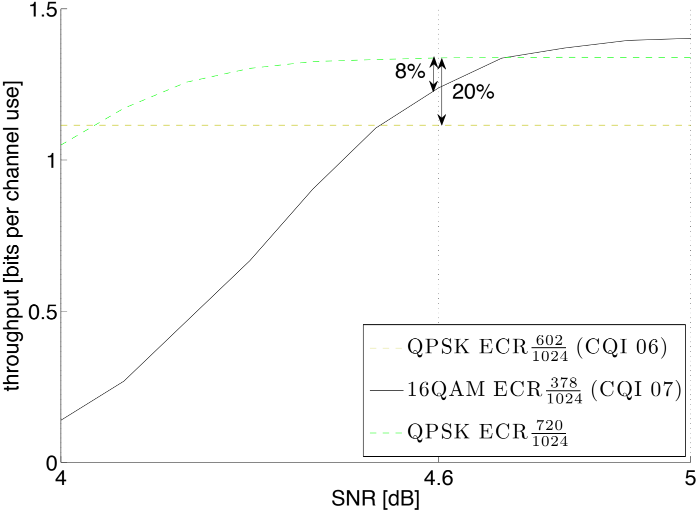
Zoom in SNR Region 4dB-5dB

According to CQI reporting rule, CQI 6 should be selected at snr 4.6dB.

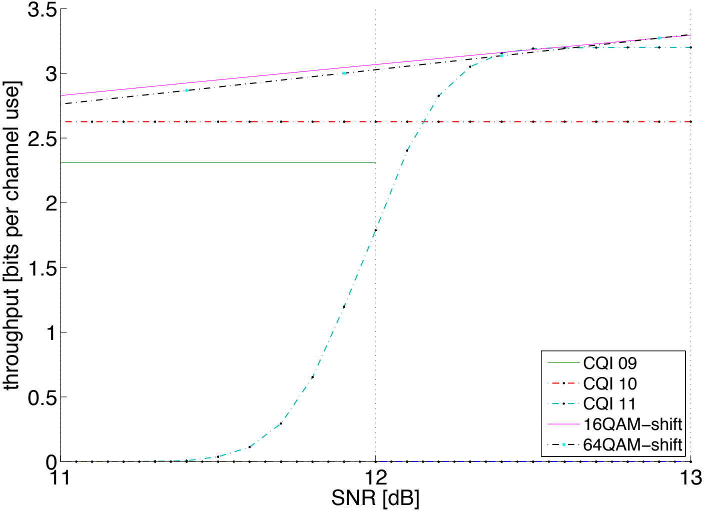


Throughput Gain of Constellation and Coding Rate by Modified Selecting Rule

QPSK and ECR $\frac{720}{1024}$ are given by modified selecting rule.

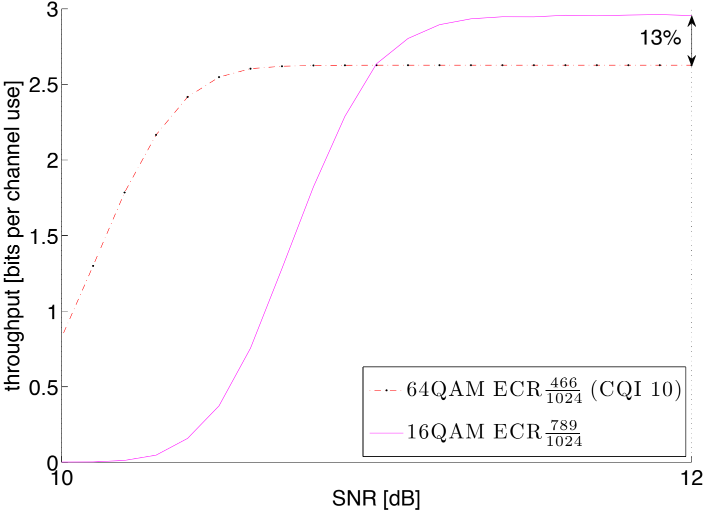


According to CQI reporting rule, CQI 10 should be selected at snr 12dB.



Throughput Gain of Constellation and Coding Rate by Modified Selecting Rule

16QAM and ECR $\frac{789}{1024}$ are given by modified selecting rule.



Conclusion

- Review of asymptotic analysis of $R_n(\epsilon)$.
- Non-asymptotic Achievability and Converse via Jar Decoding, and Non-asymptotic Equipartition Property.
- Definition of $\delta_{t,n}(\epsilon)$ and its interpretation, i.e. the relative magnitude of n and ϵ.
- Taylor Expansion of $R_{t,n}(\epsilon)$ and $R_n(\epsilon)$ with respect to $\delta_{t,n}(\epsilon)$.
- Comparison of Taylor Expansion with asymptotic analysis.
- Implication of Taylor Expansion on input distribution and practical code design.
- Numerical comparison of SO and NEP approximation derived from Taylor Expansion and Normal Approximation.
- Application of Taylor Expansion to LTE system.

Thanks.
Question?