Cross Layer Design for Network Coding Over Fading Channels

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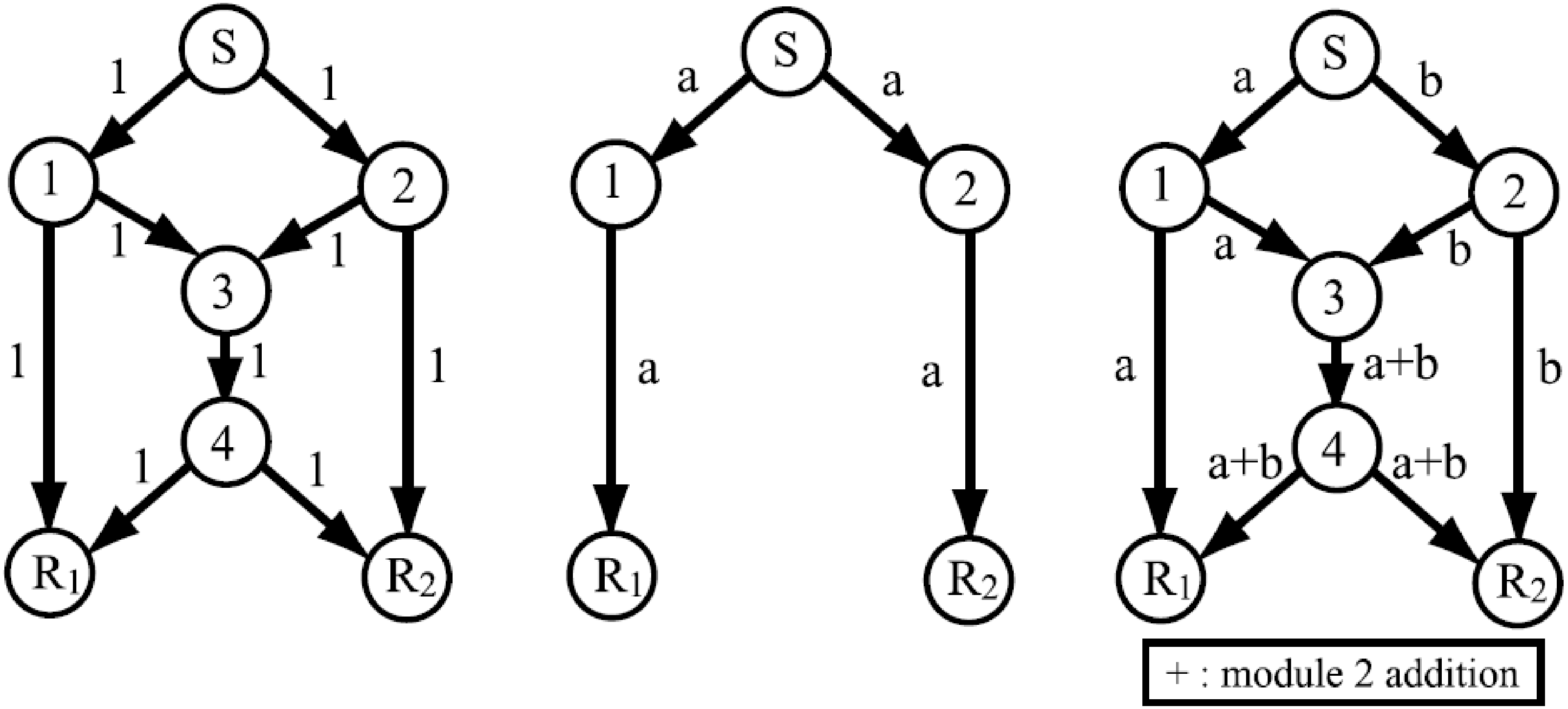
Outline

- Introduction on Network Coding
- Some Recent Key Topics
- Three-node Relay Network
- Two Variants on Three-node Relay Network

  (1) MISO Beamforming Design

  (2) Distributed Space Time Cooperation
- Summary

Network Coding Concept



(a) Link capacity (b) IP multicast (c) Network Coding

Network Coding

- 1998 HKCU Robert Li , Raymond Yeung,
  Start to Study Network Coding

- 2000 Alswede, Ning, Bob, Raymond.
  IEEE Transactions Information Theory, "Network Information Flow",

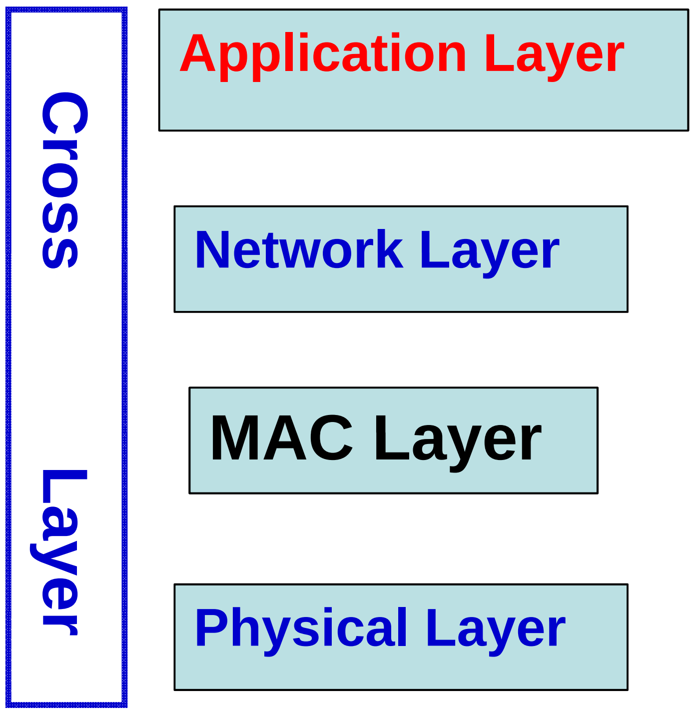
- 2002 IEEE Transactions Information Theory
  "Linear Network Coding" 2005 IT Best paper Award

- 2002, MIT, UIUC, UCLA, Caltech, Princeton, University of Maryland, Toronto etc.

- 2003 China, Tsinghua U. etc.

- 2010, more than a few hundreds Universities and Research Institutes all over the world.
- 3GPP2 Selected Network coding as its potential technology in 4G wireless systems

Network Coding Research Directions



Main topics (I)

- **Network coding at network layer**

(1) Codeword size and finite field size,

Answer: Linear code, a finite field with relatively large size.
Random linear code, Algebraic code (Matrix theory)

(2) Encoding/decoding complexity,
Coding node number in a network
Routing algorithm and encoding node finding

(3) Network coding Capacity
Maxi-Flow Min-Cut Theorem for fixed networks
Random Graph Theory for Ad Hoc Networks

(4) Secure Network Coding
Finding Security Rate Region

Main topics (II)

- Network Coding at Physical Layer
  (1) Digital Network Coding (at encoding node, bit processing or symbol processing)
  (2) Analogue Network Coding (at encoding node, signal wave superposition or mixing)

- Modulation and coding Selection
- Multi-resolution encoding/decoding
- Superposition and signal recognition /estimation /recovery
- Dirty paper coding

Main topics (III)

- Cross Layer Design

  For some simple network topologies or Ad Hoc networks with random characteristics

  Formulating some optimal problems based on certain criteria.

  (1) Increasing network coding gain

  (2) Maximizing total throughput

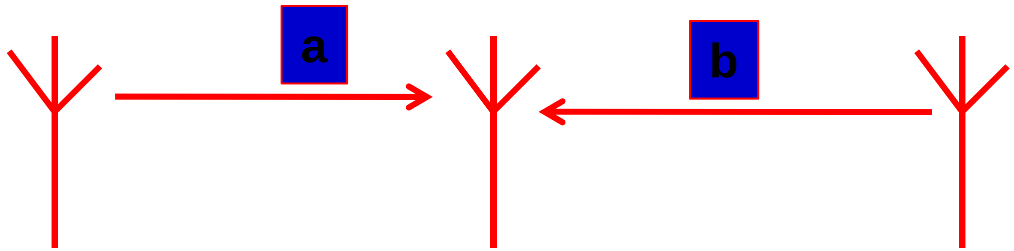
  (3) Increasing Signal to noise ratio for broadcast

  (4) Maximizing some Utility functions

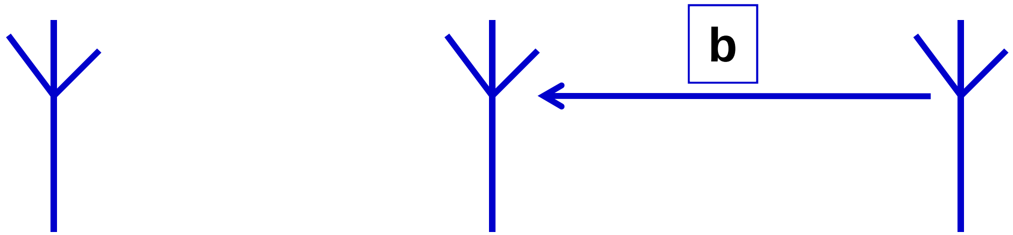
  (5) Increasing multi-user fairness

  …

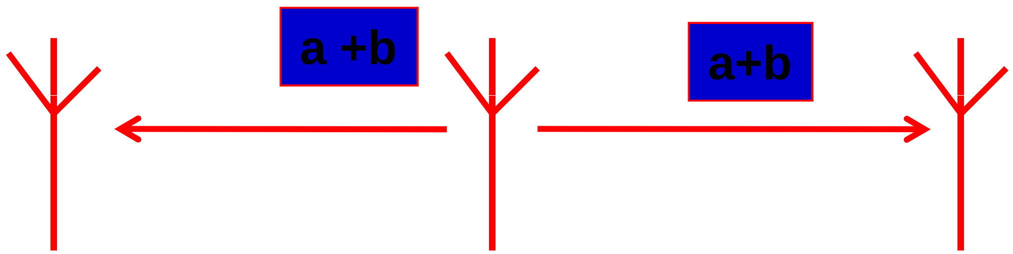
Some examples on network coding



(1)

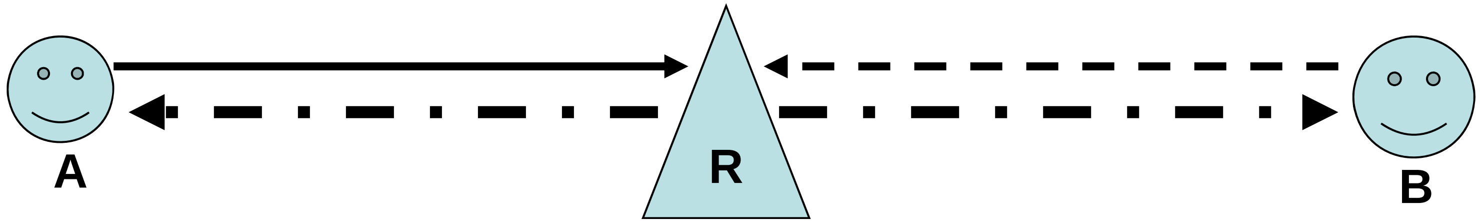


(2)

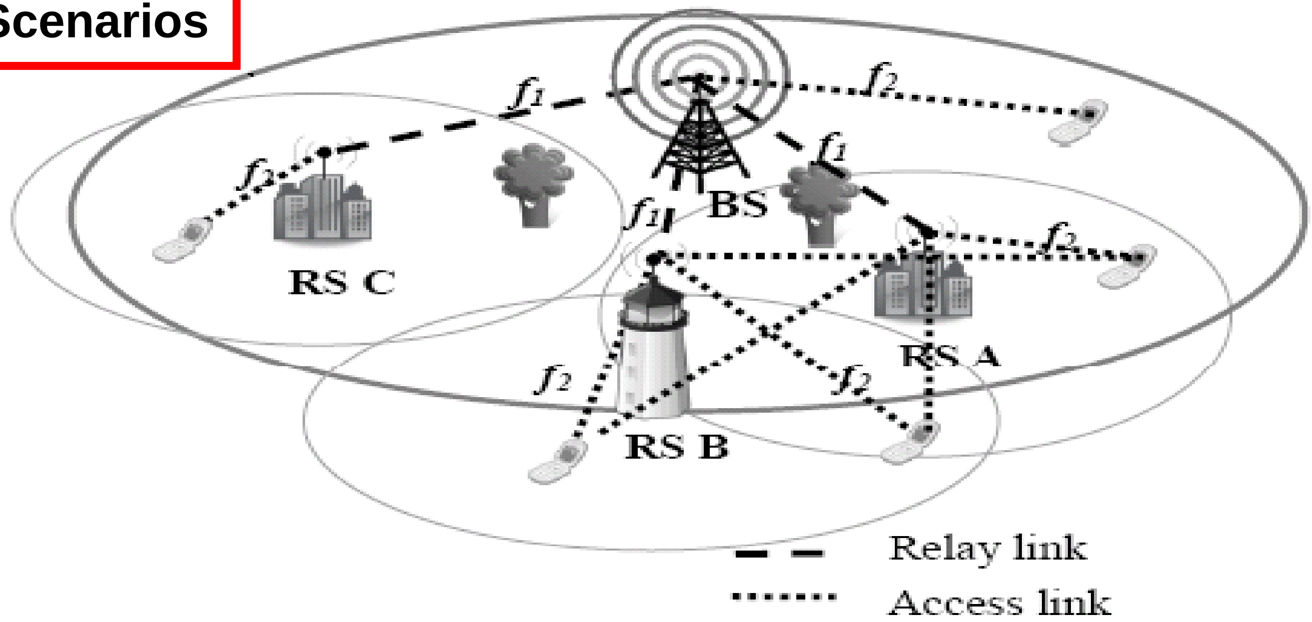


(3)

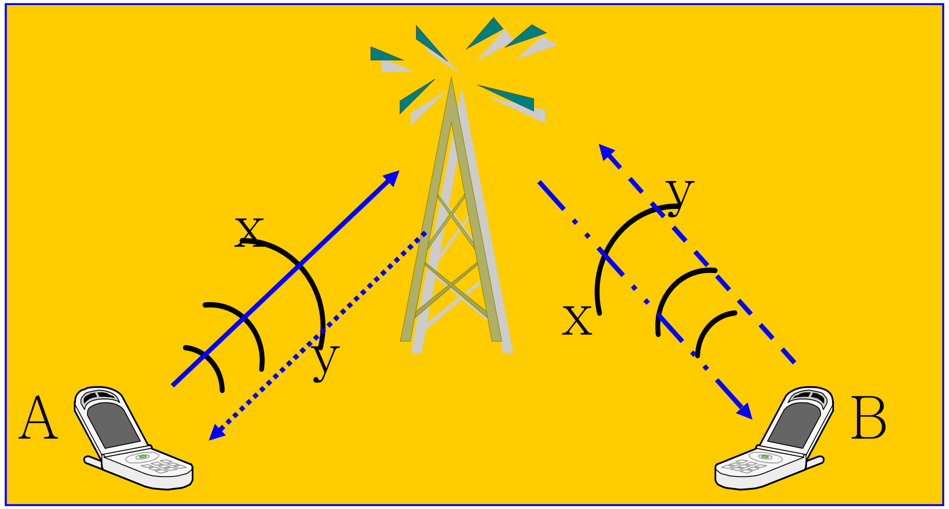
Three-node Network



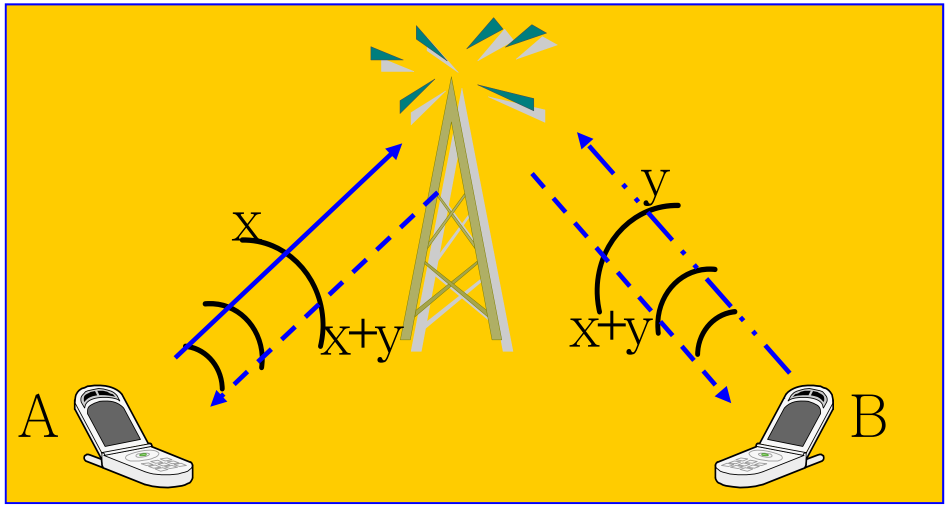
Application Scenarios



Two-way Relay network



Traditional TD Mode
Four time Slots

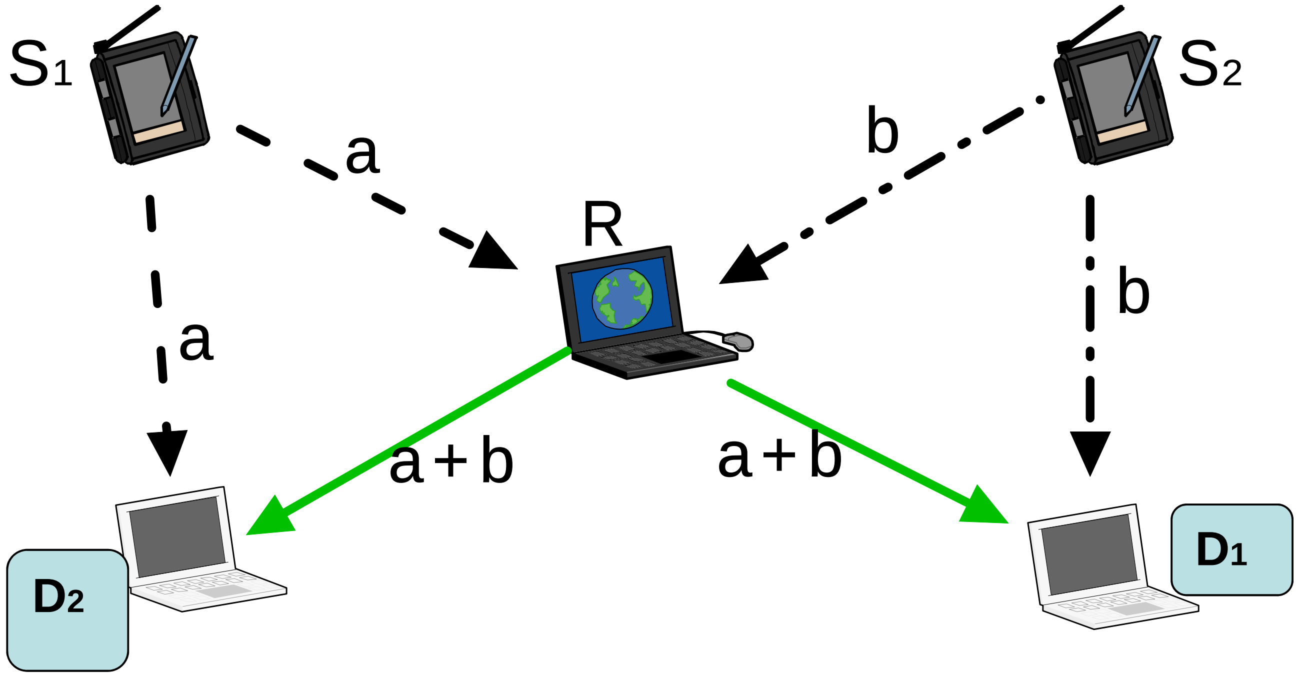


Network Coding Mode
Three time Slots

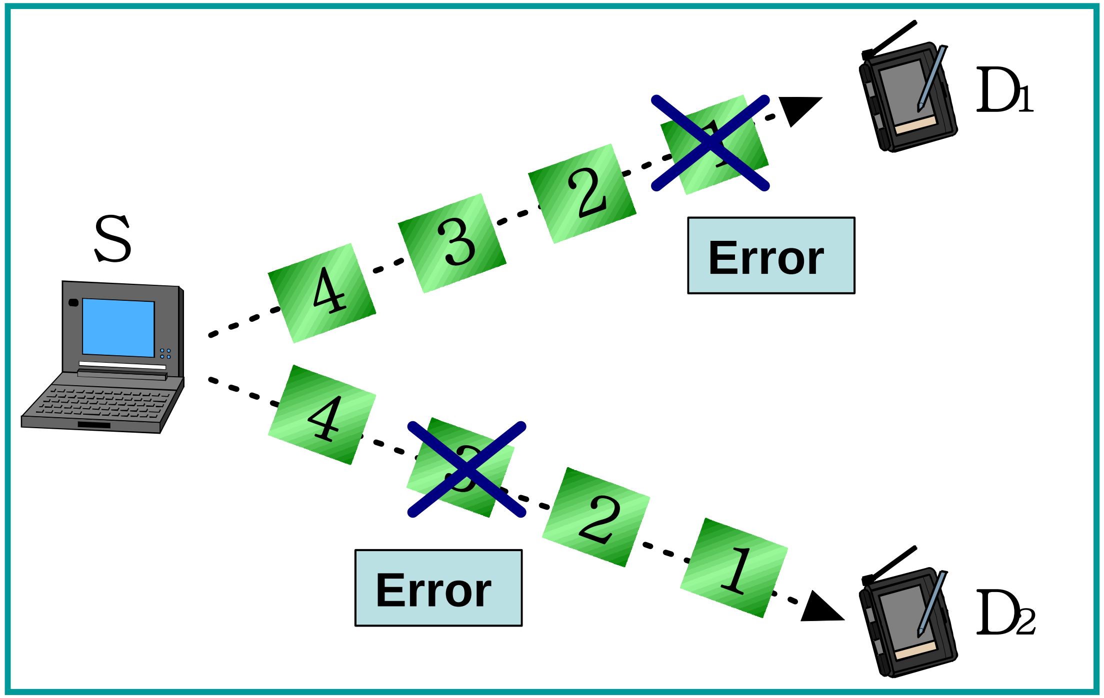
may be used to
Satellite Communications

Butterfly Network

Two-source Two-destination and one Relay



Multi-user Broadcast



(1) ARQ

(2) Network Coding +ARQ

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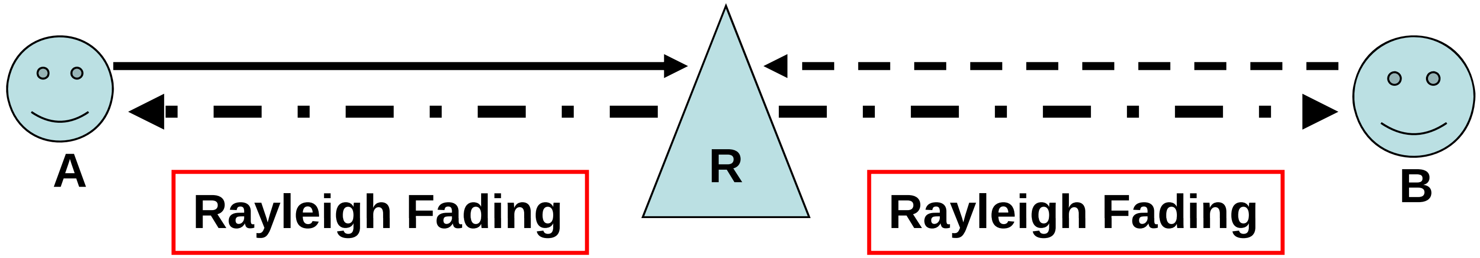
Three-node Relay Network



- What is the effect of the Channel Gain difference between these two links on Network Coding Gain?
- How to improve Network Coding Gain?
- How to Select a Relay node in the presence of many Relays ? (Proper position)

How to do?

Review the Channel fading Characteristics

- Consider a single link case

(uplink in three –node mode)

- Consider two links used for information broadcast (downlink in three-node mode)

(1) Little Channel Side Information:

Some statistics on the channels, i.e.

Fading mode, Average SNR etc.

(2) Completely Known Channel Side Information:

Instantaneous Channel fading coefficients.

Some Definitions
on Rayleigh fading Channel Characteristics

- **Received Power's Probability Density Function for Rayleigh fading**

$$f_{P_e}(\varphi) = \frac{1}{\overline{\varphi}} \cdot e^{-\varphi/\overline{\varphi}}$$

where $\overline{\varphi}$ denotes the average received power

- **Outage Probability**

$$P_{\text{out}} = \Pr(\gamma < \gamma_0)$$

- **Maximum Data Rate**

$$C_{\text{out}} = \max_{P_{\text{out}}} (1 - P_{\text{out}}) \cdot R.$$

Over Rayleigh Fading

$$P_{\text{out}} = \Pr(\gamma < \gamma_0) = \int_0^{\gamma_0} f_\gamma(\gamma)d\gamma = 1 - e^{-\gamma_0/\overline{\gamma}}.$$

$$C(R) = (1 - P_{\text{out}}) \cdot R = R \cdot e^{-\gamma_0/\overline{\gamma}}.$$

Single link
The optimization Problem:

$$\text{Maximize}: \quad C(R) = R \cdot e^{-\gamma_0/\overline{\gamma}}$$
$$\text{Subject to}: \quad R > 0$$
$$\gamma_0 = 2^{R/W} - 1.$$

Power Threshold

Downlink case: Dual Cast (Stage 3),

Little **Channel Side Information**

$$C(R) = 2R \cdot (1-P_{\text{out},1})(1-P_{\text{out},2}) + R \cdot (1-P_{\text{out},1})P_{\text{out},2} + R \cdot P_{\text{out},1}(1-P_{\text{out},2}) = R(e^{-\gamma_0/\overline{\gamma_1}} + e^{-\gamma_0/\overline{\gamma_2}}). \quad (11)$$

$$\begin{aligned} \text{Maximize} \;:\quad & C(R) = R(e^{-\gamma_0/\overline{\gamma_1}} + e^{-\gamma_0/\overline{\gamma_2}}) \\ \text{Subject to} :\quad & R > 0 \\ & \gamma_0 = 2^{R/W} - 1. \end{aligned}$$

Network Coding Constant Power Outage Capacity:

$$C_{\text{out,nccp}} = R_{\text{opt,nccp}} \cdot (e^{-\gamma_{o,\text{nccp}}/\overline{\gamma_1}} + e^{-\gamma_{o,\text{nccp}}/\overline{\gamma_2}}).$$

Two links case or Dual-Cast (Stage 3)
Full Channel Side Information

Water Filling Policy

$$P(\gamma) = \begin{cases} \frac{\sigma}{\gamma} \cdot \overline{P_s}, & \gamma \geq \gamma_0 \\ 0, & \gamma < \gamma_0 \end{cases}$$

Since the maximal average transmission power is $\overline{P_s}$, the value of σ can be given by

$$\frac{1}{\sigma} = \int_{\gamma_0}^{\infty} \frac{1}{\gamma} f_\gamma(\gamma) d\gamma.$$

Network Coding Channel Inversion Outage Capacity

Maximize :

$$C(\gamma_0) = W \log_2(1+\sigma)(e^{-\gamma_0/\overline{\gamma_1}} + e^{-\gamma_0/\overline{\gamma_2}})$$

Subject to :

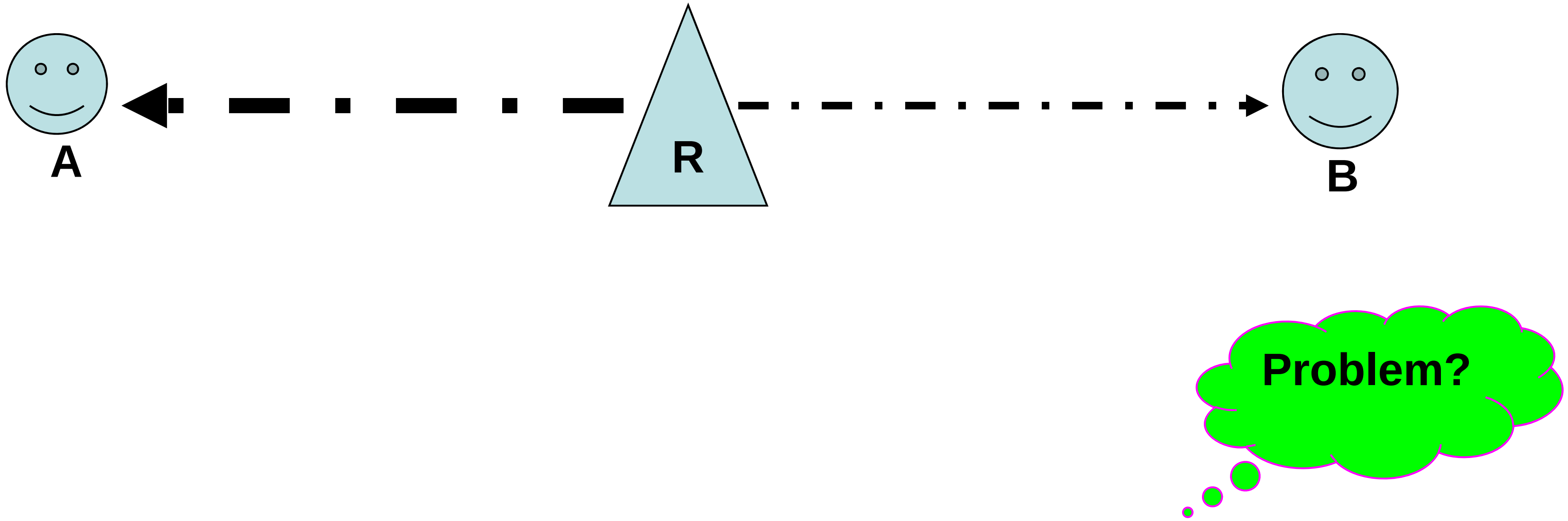
$$\gamma_0 > 0$$

$$\frac{1}{\sigma} \geq \frac{1}{\overline{\gamma_1}} E_1\left(\frac{\gamma_0}{\overline{\gamma_1}}\right) - \int_{\gamma_0}^{\infty} \frac{e^{-\gamma_2/\overline{\gamma_2}}}{\overline{\gamma_2}} \frac{1}{\overline{\gamma_1}} E_1\left(\frac{\gamma_2}{\overline{\gamma_1}}\right) d\gamma_2$$

$$+ \frac{1}{\overline{\gamma_2}} E_1\left(\frac{\gamma_0}{\overline{\gamma_2}}\right) - \int_{\gamma_0}^{\infty} \frac{e^{-\gamma_1/\overline{\gamma_1}}}{\overline{\gamma_1}} \frac{1}{\overline{\gamma_2}} E_1\left(\frac{\gamma_1}{\overline{\gamma_2}}\right) d\gamma_1 .$$

Some numerical results

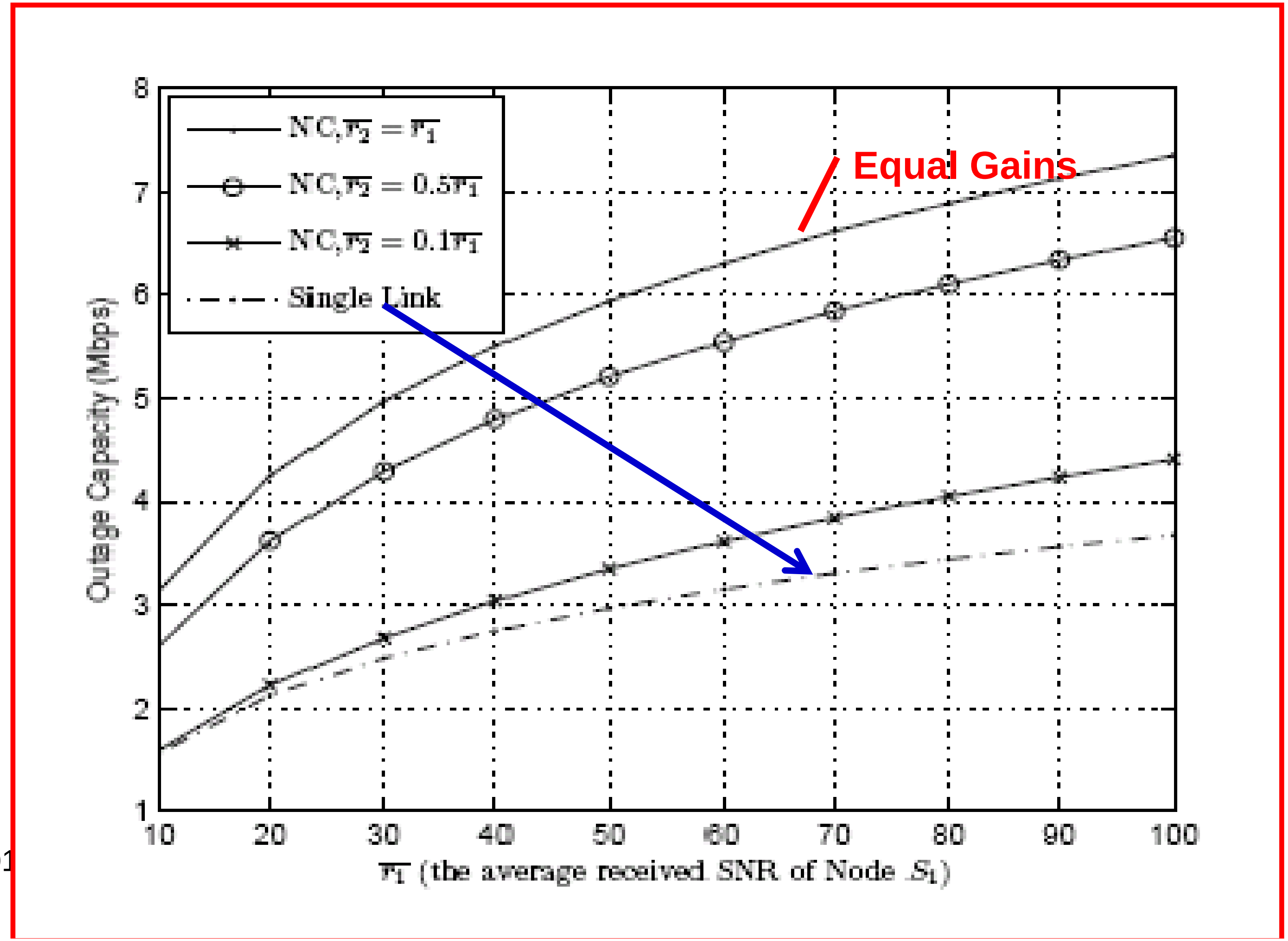
For Downlink transmission



What is the effect of the difference of channel gains over these two links on Network Coding Gain?

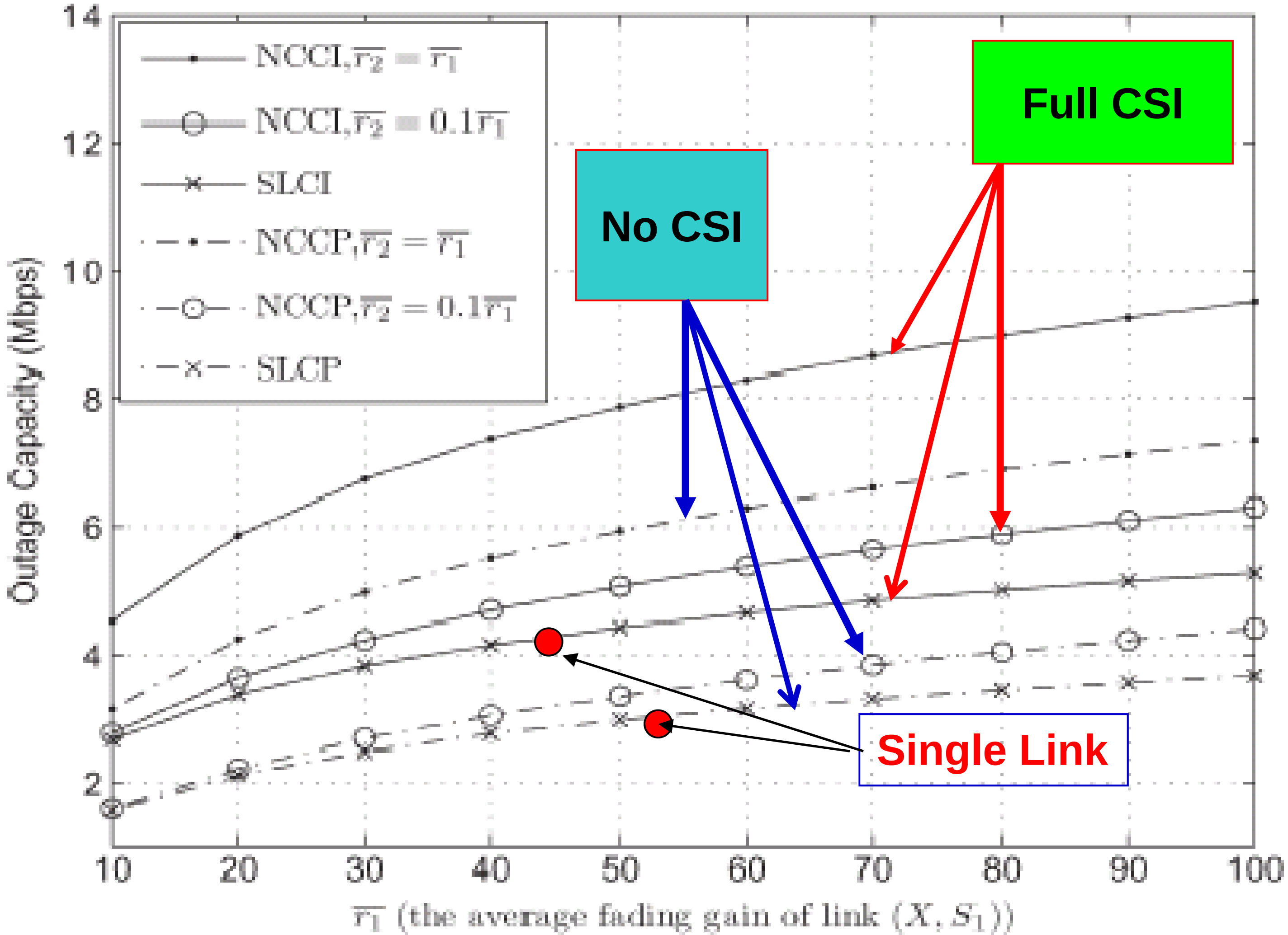
Outage Capacity,

Difference between Channel Gains on Outage Capacity



Outage Capacity Comparison

for two different NC Modes



Two Observations:

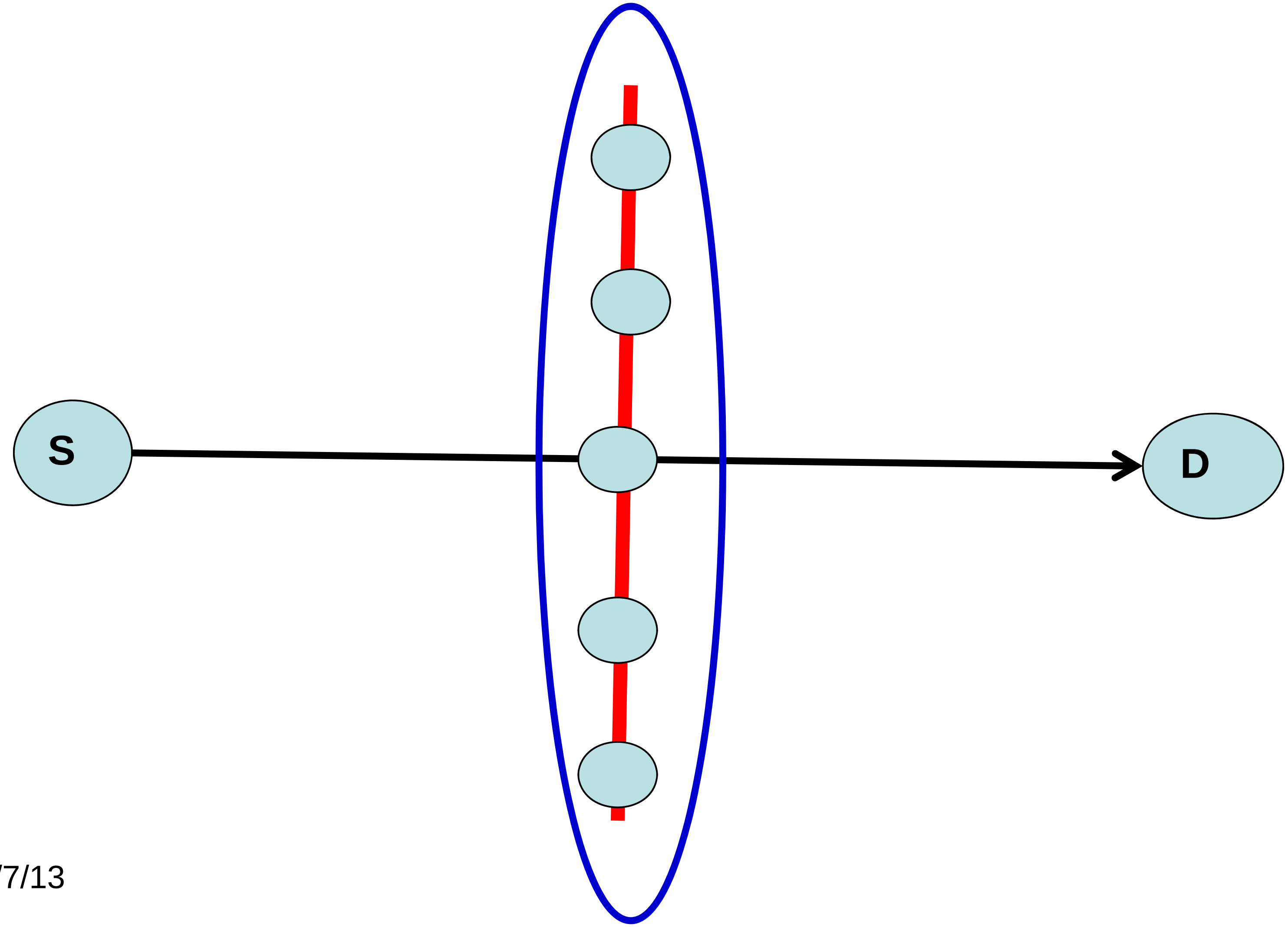
- **If the difference of Channel Gains is relatively small, the network coding gain will be close to 2. If the difference of channel gains is very large, the network coding gain will disappear.**

- **Using network coding will get a certain coding gain. Full CSI will provide more throughput than without CSI.**

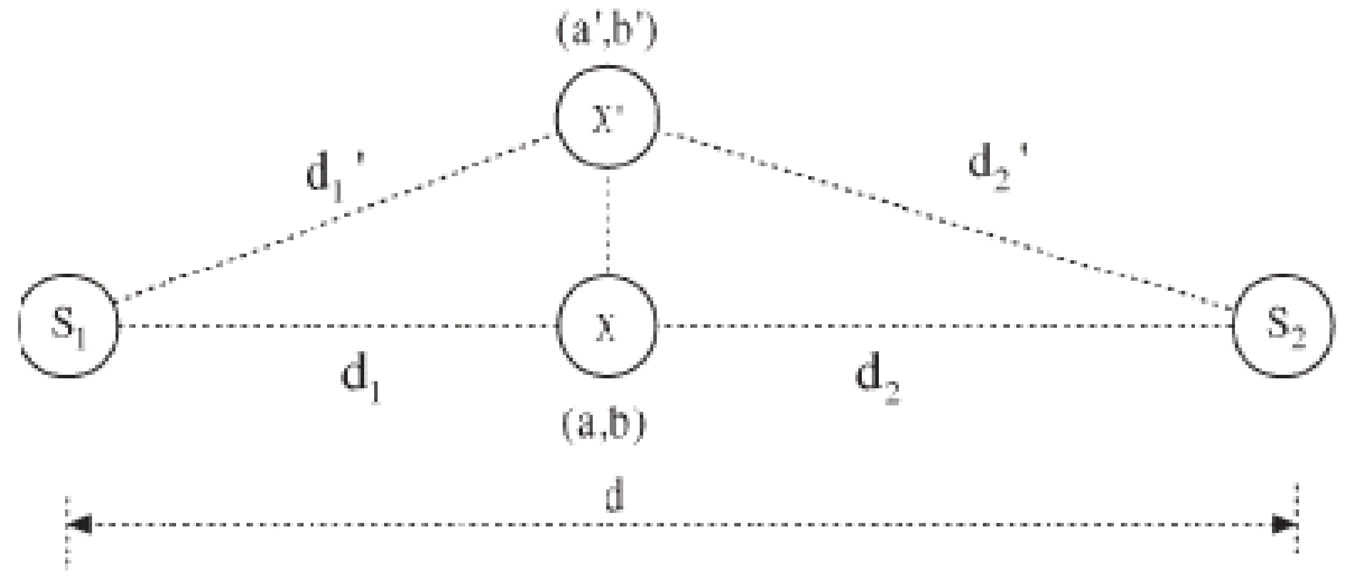
One Suggestion:

- Multi-antenna, or Relay position selection are required, Which will increase the system throughput or network coding gain

First Way: On the Relay Position Selection



How to Select Relay Position



Theorem 1: Assuming that the average channel gain of link (X, S_i) is $\overline{g_i} = \beta \cdot d_i^{-\alpha}$, $i = 1, 2$, the NCCP outage capacity is maximized, if nodes X, S_1, and S_2 are on a straight line and $d_1 = d_2$, where d_i denotes the distance between the relay node X and receiver S_i.

Second Way: On the Multi-antenna Processing

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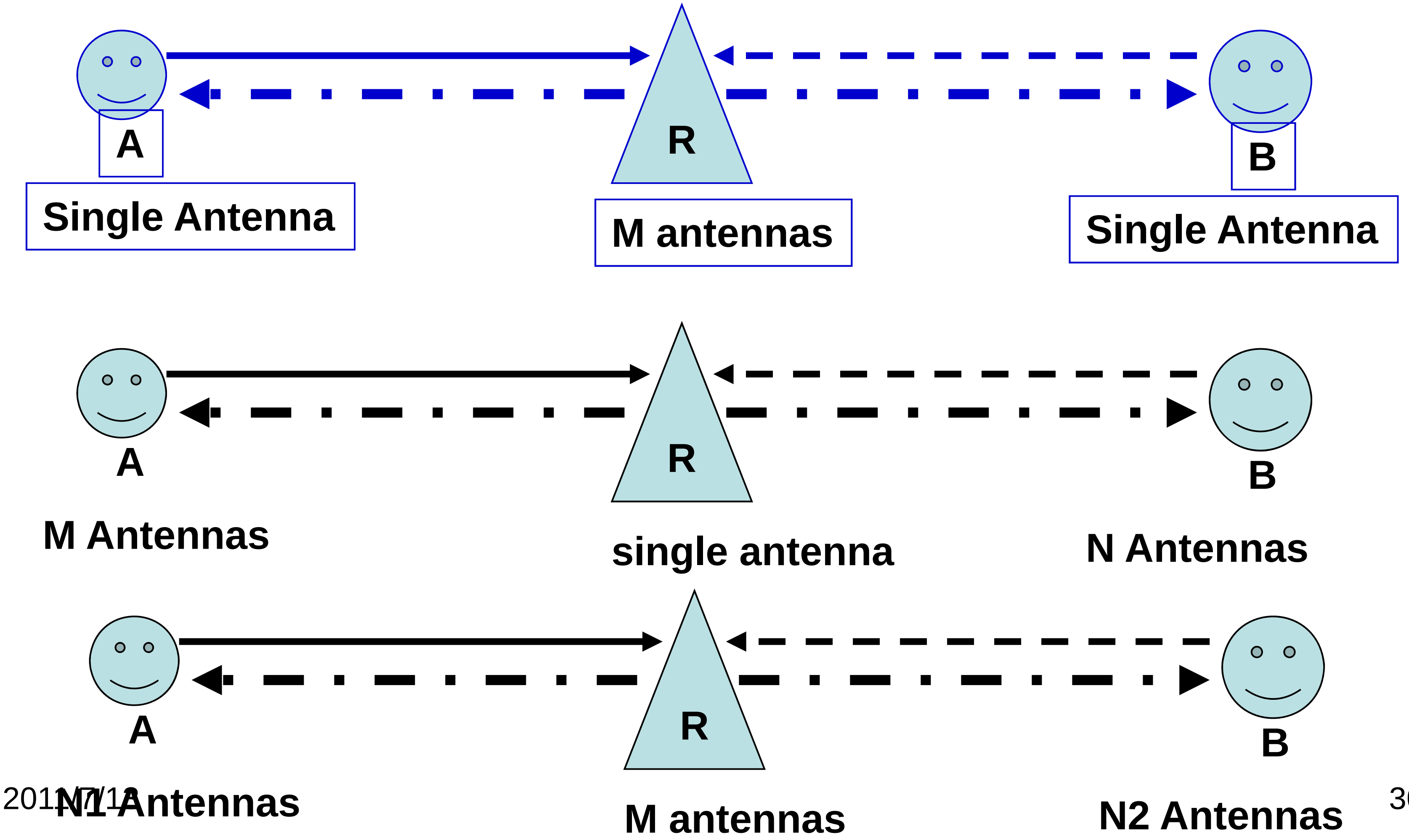
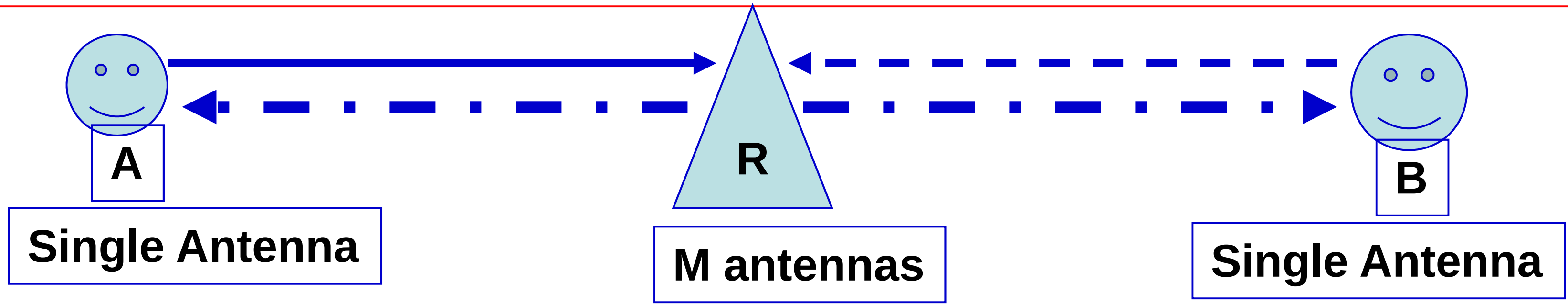
One Variant on Three-node Relay Network

There are three different cases:



MISO on Three-node Relay Network

Two difference Cases

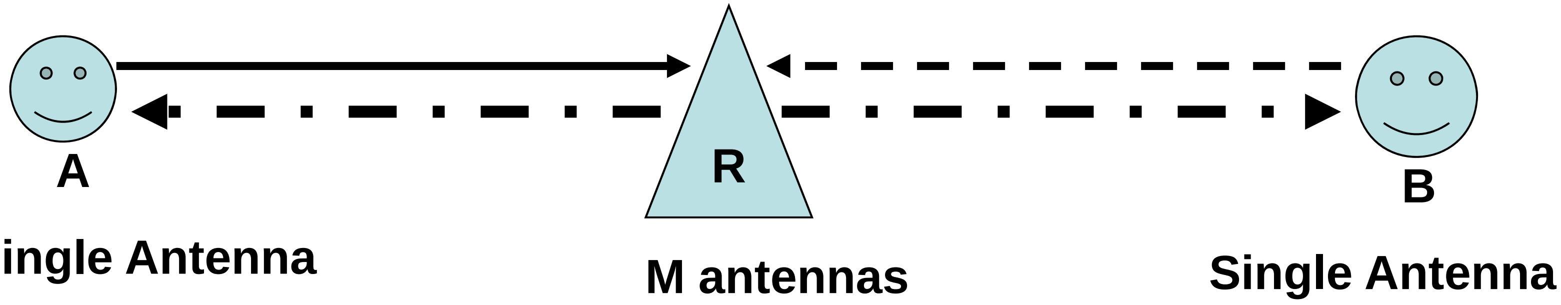


Stage 3, Employing Beam-forming:

Need to design beam-forming,
How to select the weighting factor for multi-antenna W?

- Simple Case: Full CSI
- Practical Case: Estimated CSI with bias

Full CSI Case



Stage 3, Problem :

Need to design beam-forming,
How to select the weighting factor W?

The received signal

$$y_i = \sum_{j=1}^{L} h_{ij} \cdot w_j \cdot s + n_i, \qquad i = 1, 2,$$

$$y_i = (\mathbf{h}_i{}^T \mathbf{w}) s + n_i,$$

Optimal Problem for downlink

The normalized SNR for destination *i*

$$\gamma_i = |\mathbf{h}_i{}^T \mathbf{w}|^2 E_s$$

Max-flow Min-cut Theorem:

$$R \leq \log_2(1 + \min\{\gamma_1, \gamma_2\})$$

Simple Case: Full Channel Side Information:

$$\begin{array}{ll} \max_{\mathbf{w}} & \min\{|\mathbf{h}_1{}^T \mathbf{w}|^2, |\mathbf{h}_2{}^T \mathbf{w}|^2\} \\ s.t. & \|\mathbf{w}\|^2 \leq P. \end{array}$$

Some Useful Results

Lemma 1: The optimal beamforming vector $\mathbf{w}^*$ is in the space spanned by the complex conjugates of users' channel vectors, i.e. $\mathbf{w} = \lambda_1 \overline{\mathbf{h}}_1 + \lambda_2 \overline{\mathbf{h}}_2$, where $\overline{\mathbf{h}}_i$ denotes the complex conjugate of channel vector $\mathbf{h}_i$, λ_1 and λ_2 are complex constants.

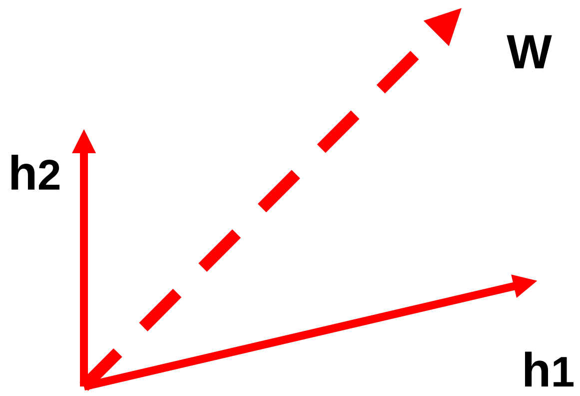


illustration on the weighting factor

Main Result

Simple Case

Theorem 1: When the users' channel vectors $\mathbf{h}_1$ and $\mathbf{h}_2$ are orthogonal (i.e. $c_{12} = c_{21} = 0$), the optimal coefficient λ_1^* and λ_2^* should satisfy

$$|\lambda_1| = \sqrt{\frac{c_2 P}{c_1^2 + c_1 c_2}}, \tag{9}$$

and

$$|\lambda_2| = \sqrt{\frac{c_1 P}{c_2^2 + c_1 c_2}}, \tag{10}$$

which will lead to the maximum of the objective function as

$$\gamma_1 = \gamma_2 = \frac{c_1 c_2 P}{c_1 + c_2}. \tag{11}$$

where c_i, $i = 1, 2$, stands for $\mathbf{h}_i^H \mathbf{h}_i = \|\mathbf{h}_i\|^2$, c_{12} for $\mathbf{h}_2^H \mathbf{h}_1$

General Case

Orthogonal Generalization →

$$e_1 = \overline{h}_1$$

$$e_2 = \overline{h}_2 - \frac{{h_2}^H h_1}{c_1}\overline{h}_1.$$

$$\|e_1\|^2 = c_1, \qquad \|e_2\|^2 = c_2 - \frac{|c_{12}|^2}{c_1}.$$

$$y_1 = {h_1}^T w s + n_0 = \mu_1 c_1 s + n_0,$$

$$y_2 = ({h_2}^T w)s + n_0 = \mu_1({h_1}^H h_2)s + \mu_2\|e_2\|^2 s + n_0.$$

Optimal Problem Formulation

$$\max_{\mu_1,\mu_2} \quad \min\{|\mu_1|^2 c_1^2, |\mu_1 \mathbf{h}_1{}^H \mathbf{h}_2 + \mu_2 \left(c_2 - \frac{|c_{12}|^2}{c_1}\right)|^2\}$$

$$s.t. \quad |\mu_1|^2 c_1 + |\mu_2|^2 \left(c_2 - \frac{|c_{12}|^2}{c_1}\right) \leq P.$$

Theorem 3: If $|c_{12}|$ is smaller than both c_1 and c_2, the objective function achieves its maximum when

$$|\mu_1| = \sqrt{\frac{(c_1 c_2 - |c_{12}|^2)P}{c_1^2(c_1 + c_2 - 2|c_{12}|)}}, \tag{20}$$

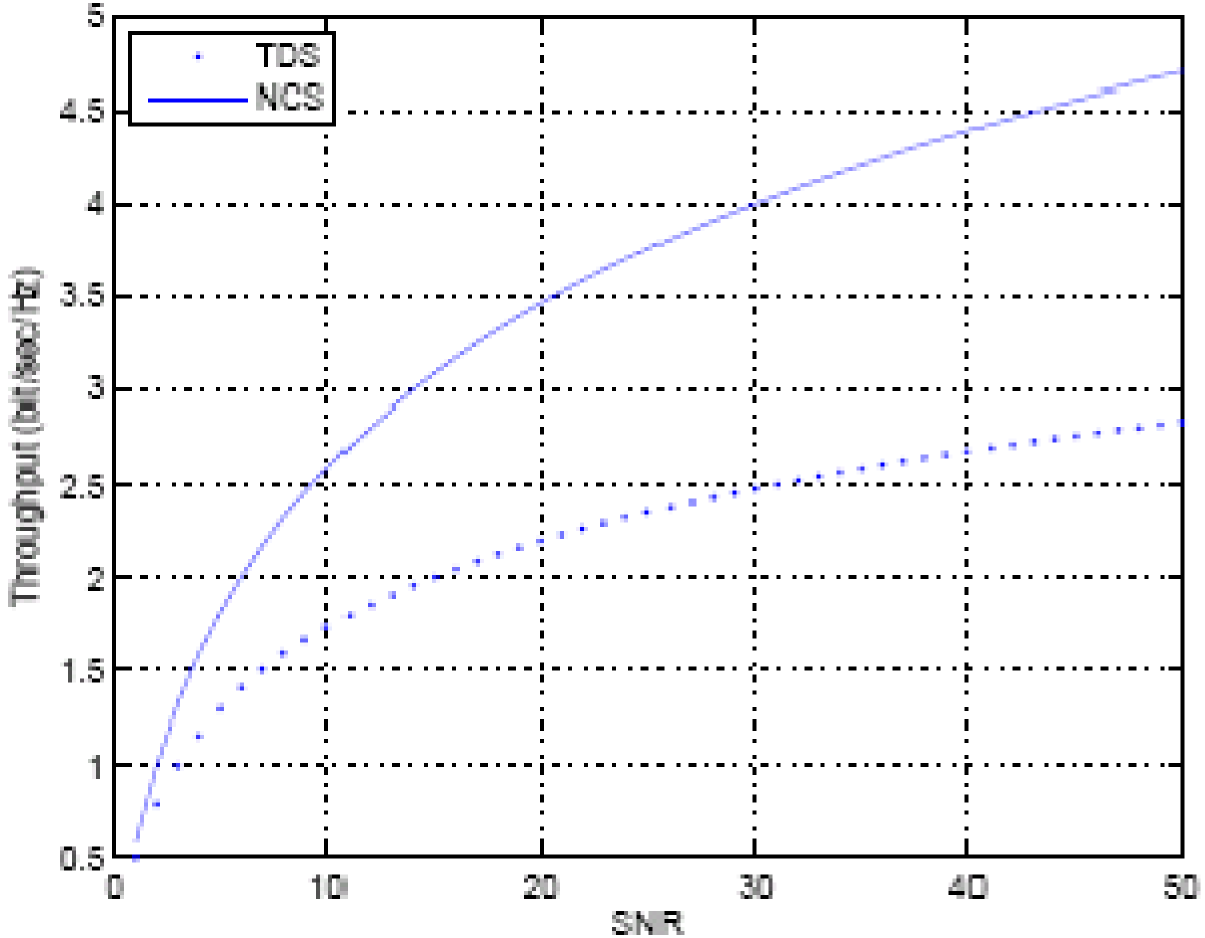
and

$$|\mu_2| = \sqrt{\frac{(c_1 - |c_{12}|)^2 P}{(c_1 + c_2 - 2|c_{12}|)(c_1 c_2 - |c_{12}|^2)}}. \tag{21}$$

which leads to the resulting maximum

$$\gamma_1 = \gamma_2 = \frac{c_1 c_2 - |c_{12}|^2}{c_1 + c_2 - 2|c_{12}|} P. \tag{22}$$

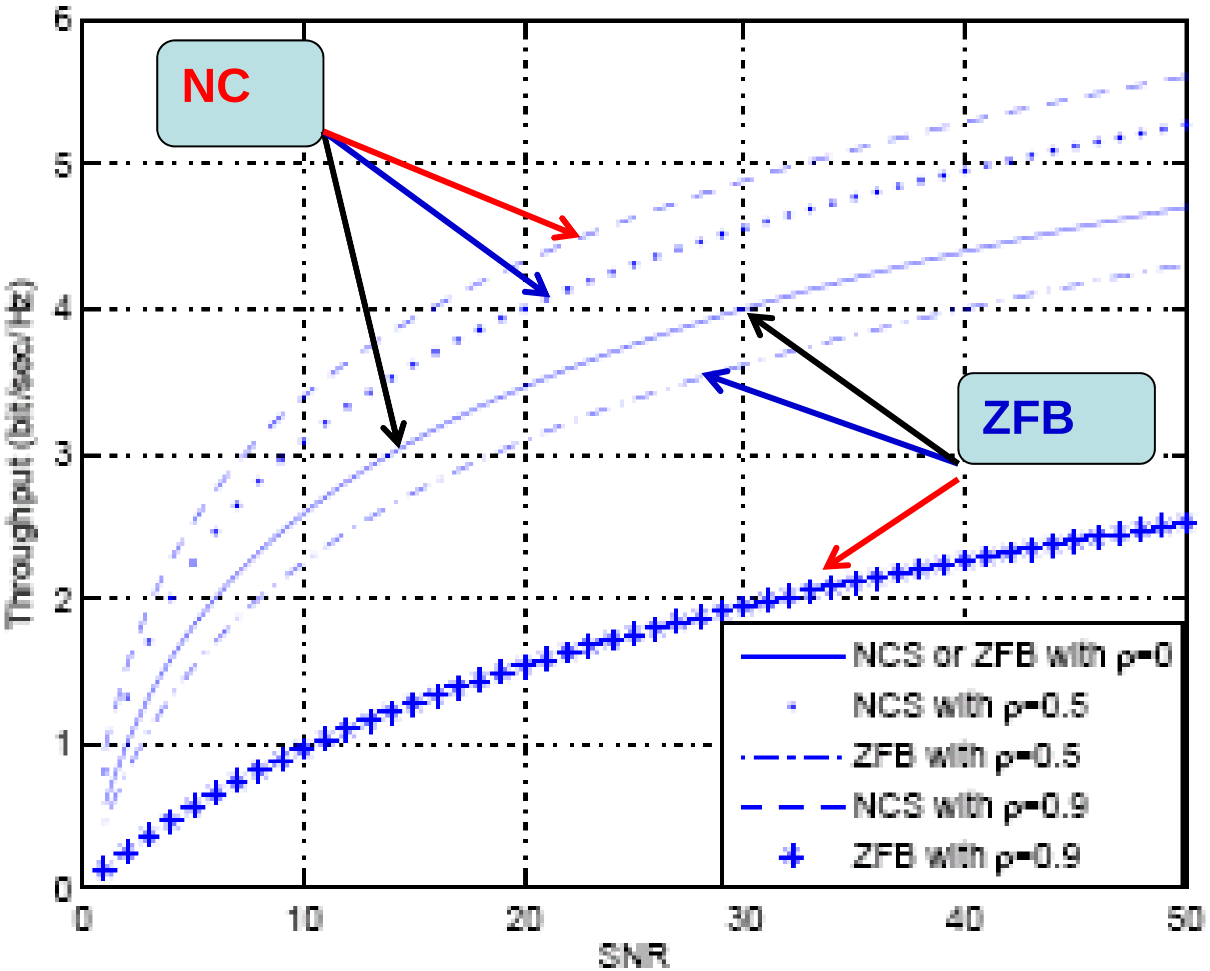
Comparison with TD Mode for downlink



TD:

- **Part of subslots used for D1**
- **Remained subslots used for D2**

Comparison with Zero-forcing Mode



The coefficient is larger,
the NC gain is larger,
ZFB gain is worse

where \rou is the coefficient of the two channel gains

Practical Case: Estimated CSI with bias

Consider Channel estimation error

$$\hat{\mathbf{h}}_i = \mathbf{h}_i + \epsilon_i \,,$$

Two Ways to solve the beam-forming design:

(1) Using the estimated CSI to replace Full CSI

(2) Finding other ways to start new design

Estimated CSI with bias

Consider Channel estimation error

$$\hat{\mathbf{h}}_i = \mathbf{h}_i + \epsilon_i ,$$

New Proposed Weighting Mode

$$\max_{\mathbf{w}} \quad \min\{|\alpha_1 \hat{\mathbf{h}}_1^T \mathbf{w}|^2, |\alpha_2 \hat{\mathbf{h}}_2^T \mathbf{w}|^2\}$$
$$s.t. \quad \|\mathbf{w}\|^2 \le P,$$

$$\alpha_1 = \frac{G_1}{G_1 + \sigma_{\varepsilon,1}^2}$$

$$\alpha_2 = \frac{G_2}{G_2 + \sigma_{\varepsilon,2}^2}$$

Main Result on outage probability

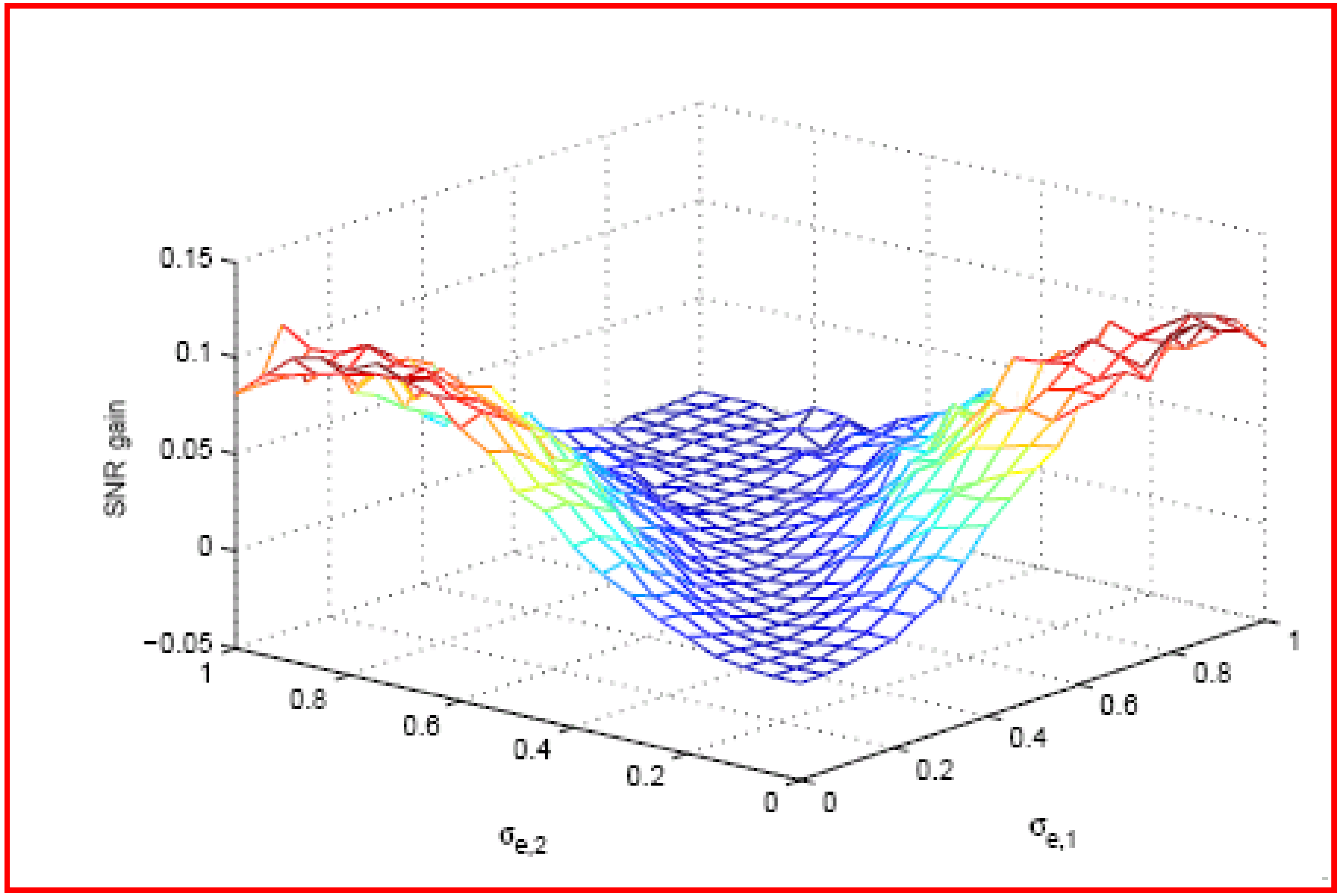
Theorem 2: The outage probability of user i is upper bounded by

$$P_{out,i}(\beta) \leq (\exp(-\frac{(b_i-\beta)^2}{\sigma'^2_{e,i}}) - \exp(-\frac{(b_i+\beta)^2}{\sigma'^2_{e,i}}))\frac{1}{\pi}\arcsin\frac{\beta}{b_i}$$

Note That \beta is a predefined threshold

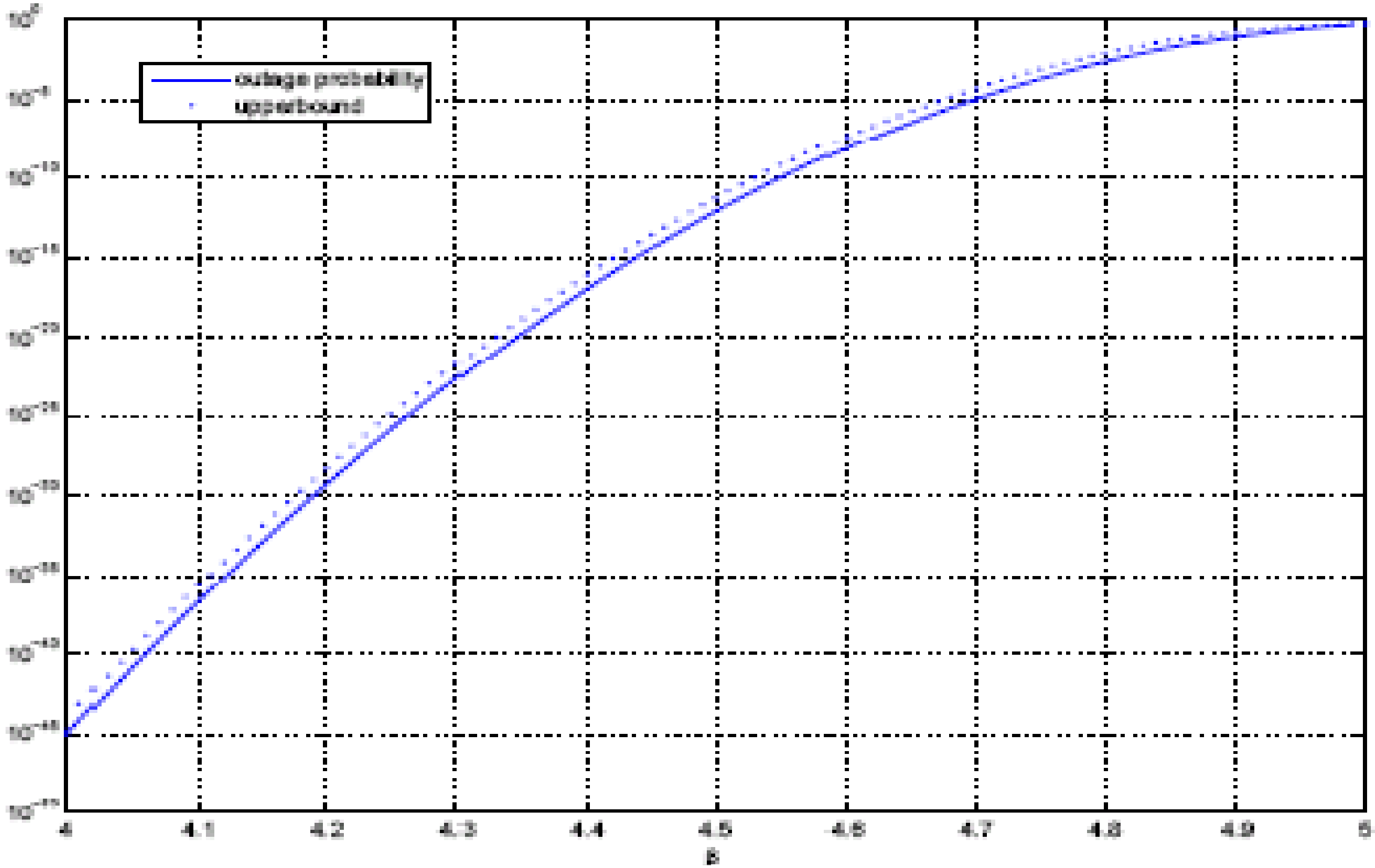
Beam-forming Gain with Estimation Errors

Two different ways



Outage Probability

Outage Probability



Parameter \beta

Different Ways

To deal with the network coding Application

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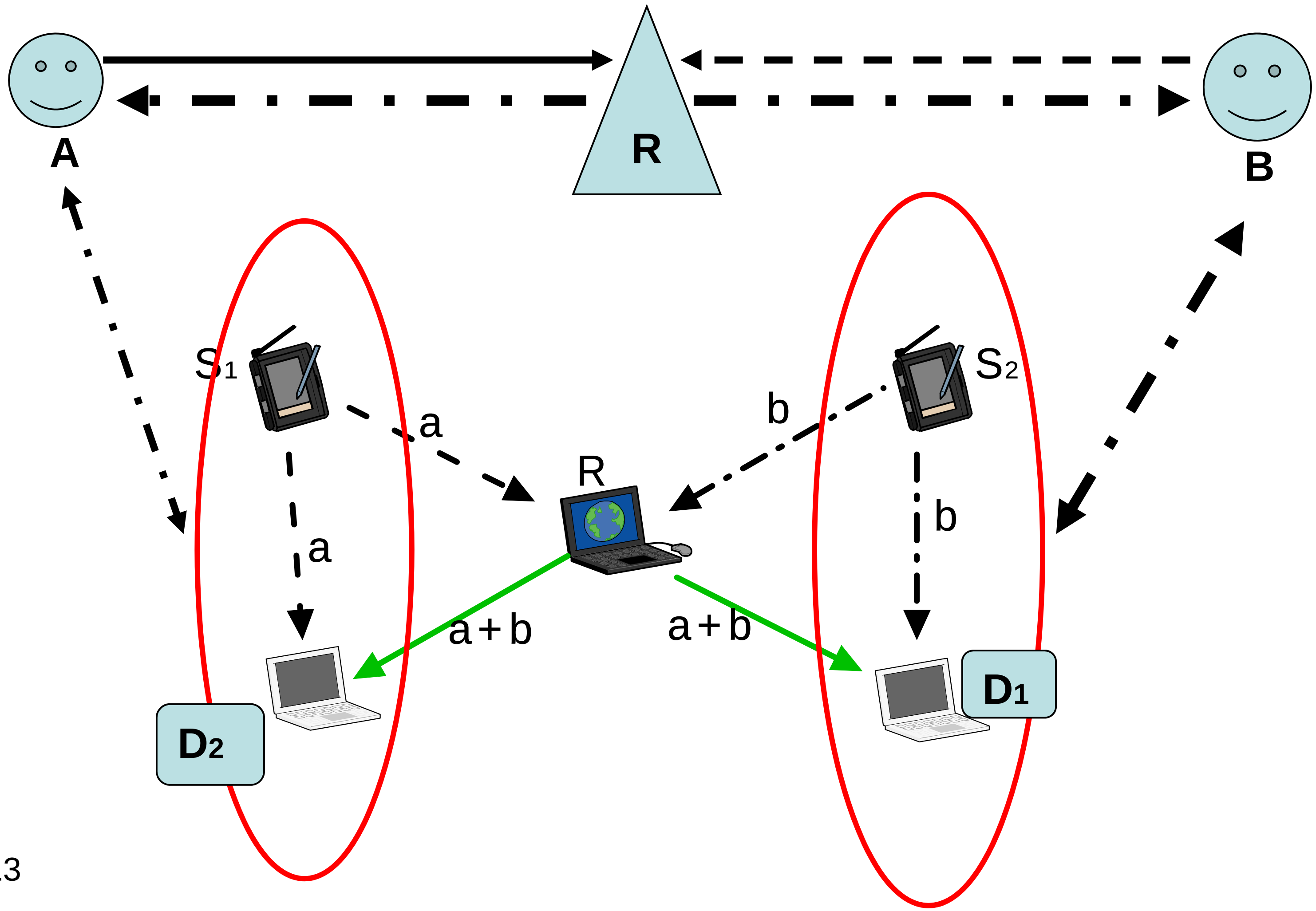
Second Variant on Three-node Relay Network



Distributed Space time Cooperation+ONC

- ARQ with limited times
- D2 and S1 ----Almouti Space Time coding

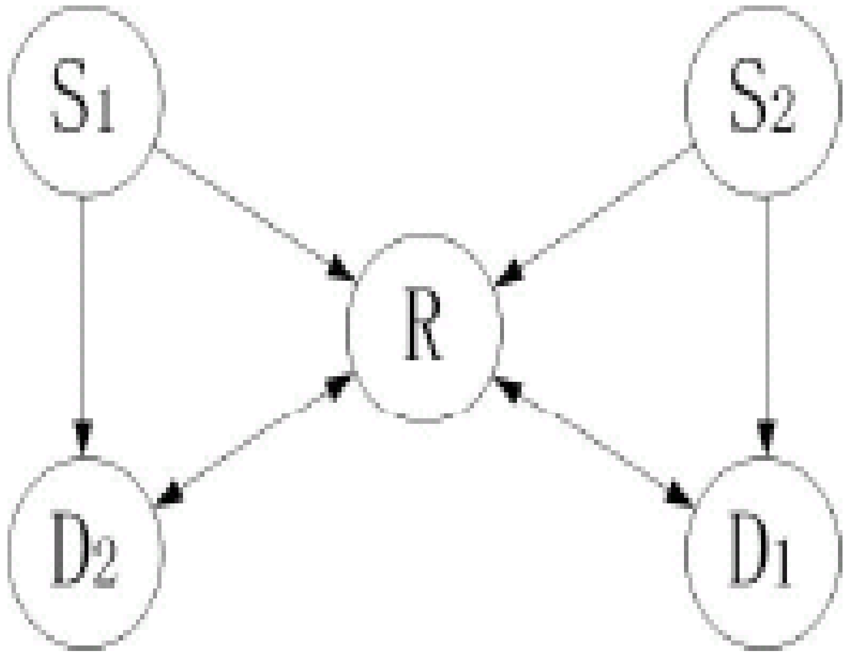
- ARQ with limited times
- D1 and S2 ----Almouti Space Time Coding

- Repeat Transmissions and Opportunistic Network Coding

Three Indices

(1) Throughput
(2) Packet Delay
(3) Network Coding Gain

Distributed Space time Cooperation+ ONC



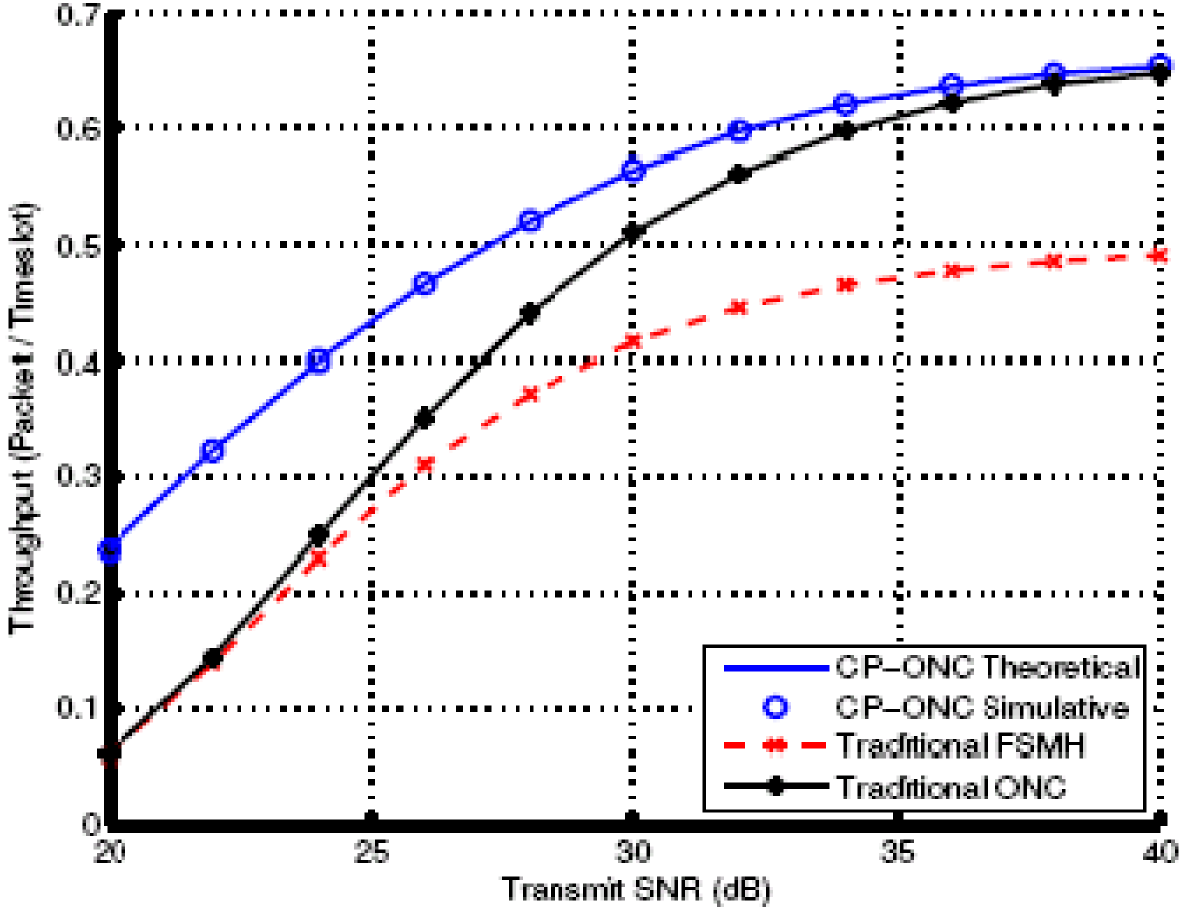
(1) Four Time Slots Transmission

(2) Opportunistic Network Coding

(3) Distributed Space Time Cooperation + ONC

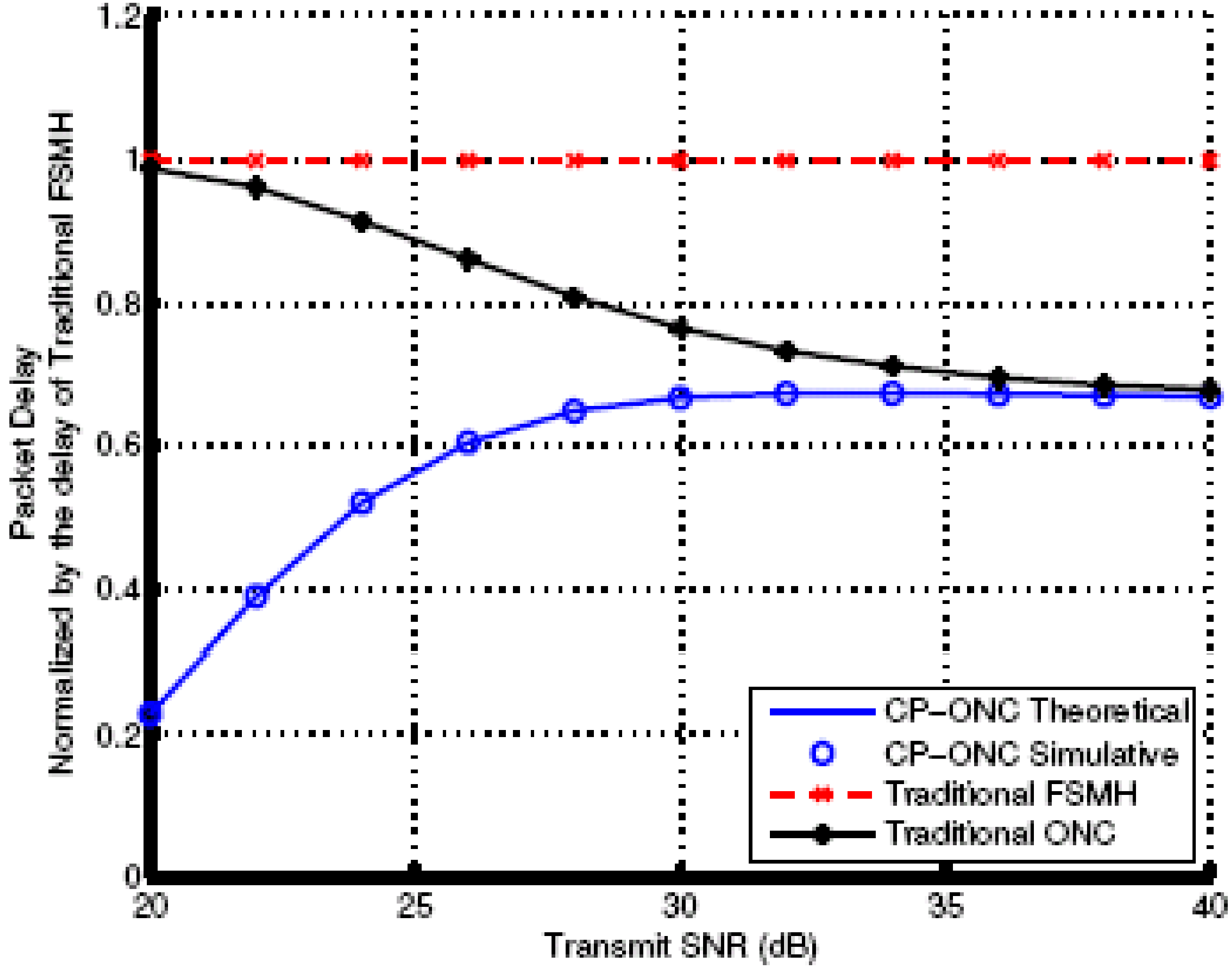
Butterfly Network with Some Modifications

Throughput Comparison

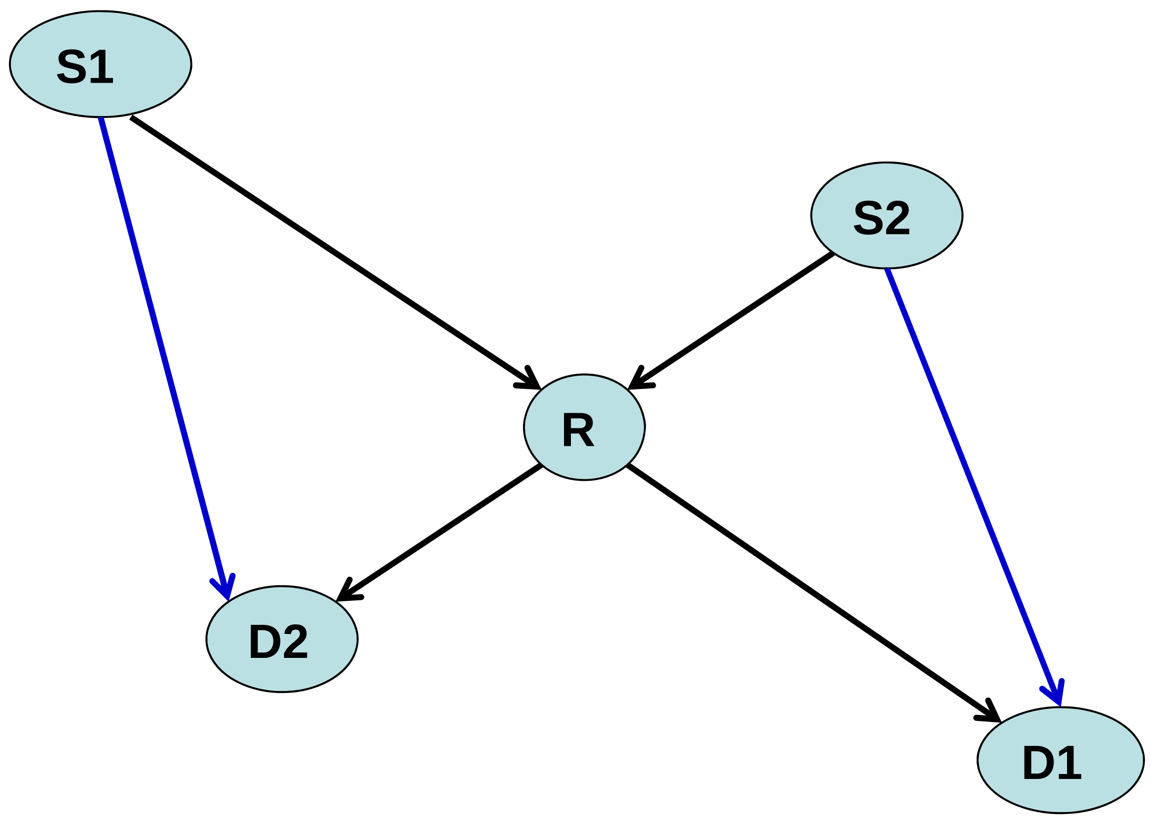


Every node uses the same transmit power

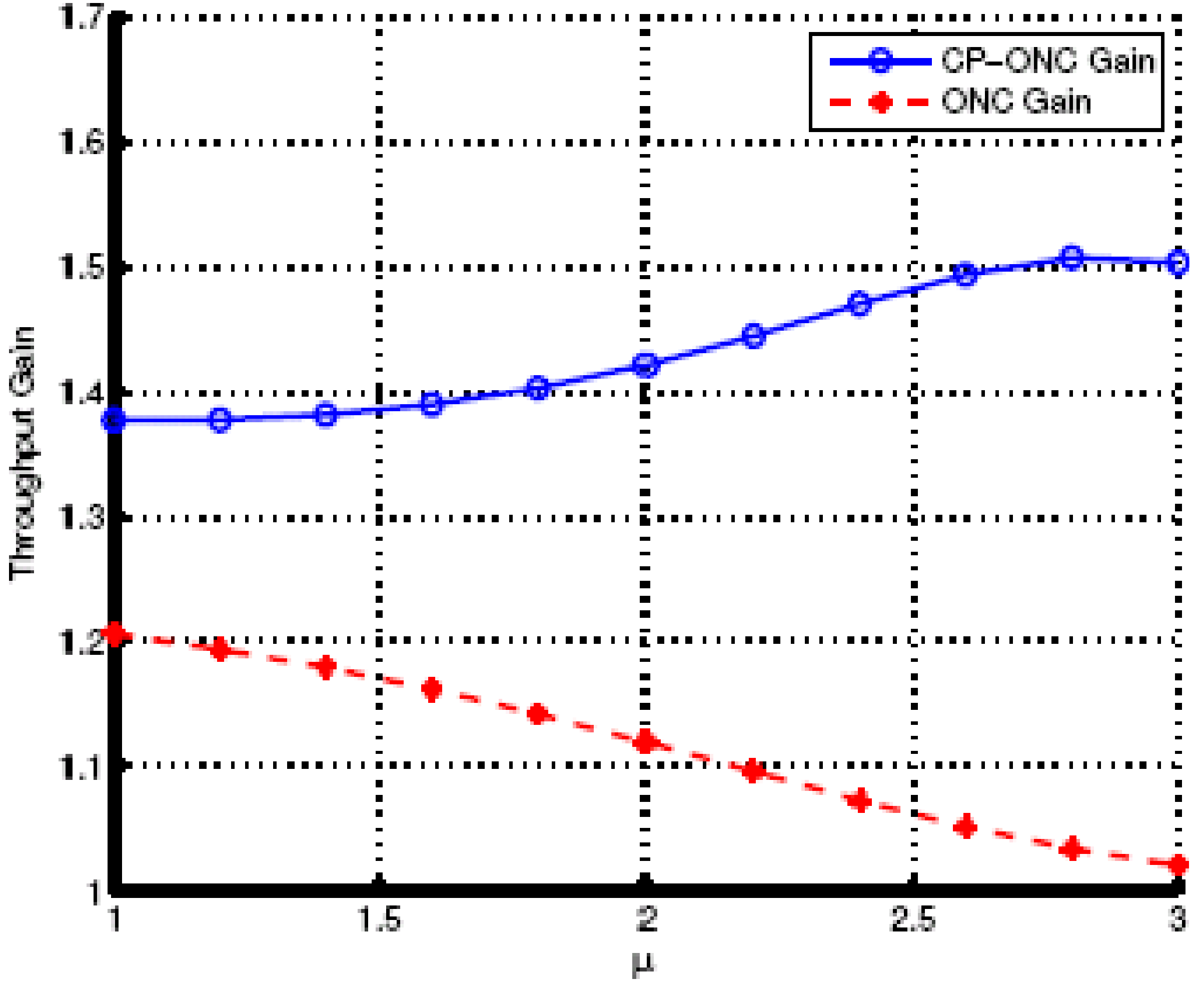
Packet Delay Comparison



Topology variation



Throughput Gain via Distance Ratio



The distance ratio between D1-R and D2-R

What the effect of Physical Topology variation on Coding Gain

Compared with D F (Four time slots) Transmission Mode

Main Observations

- Low SNR, Distributed Space time Cooperation + ONC provides considerable coding gain.
- High SNR, the coding gain of Distributed Space time Cooperation + ONC will close to that of ONC.
- Distributed Space time Cooperation + ONC has a better robust to the distance ratio variation or **the physical topology variation**.

Summary

- **Show how to do the cross layer design on Network Coding by three examples.**
- **Network coding will provide some coding gains in some cases, but not always.**
- **Optimal Problem formulations are based on some resource constraints. Here we employed maximum transmission rate with outage probability.**

  Other Utility Functions may be good selections.

Related Publications

- Jingyi Hu, **Pingyi Fan**, Ke Xiong,*y*, Su Yi and Ming Lei, "Cooperation-based Opportunistic Network Coding in Wireless Butterfly Networks" ***Accepted by IEEE Globecom2011, July, 2011.***

- Zhengfeng Xu, Hong-Chuan Yang, **Pingyi Fan**, Su Yi, Ming Lei, "Optimal Dual-cast Beamforming for Network Coding-based Two-way Relay Transmission" ***IEEE IWCMC2011, July, 2011.***

- Wei Li,Jie Li, **Pingyi Fan**, "Optimal Date Rate and Opportunistic Scheme on Network Coding Over Rayleigh Fading Channels" ***IEEE MSN2010, May, 2011.***

- Wei Li, Jie Li, **Pingyi Fan,** " Network coding for two-way relaying network over Rayleigh fading channels" ***IEEE Trans. Vehicular Technology, Vol.59, no.9, Nov., 2010. pp.4476-4488.***

Thanks!

Question?