Multiple Descriptions, Gaussian Source Broadcast and Source-Channel Separation

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Based on joint work with Jun Chen, Suhas Diggavi, Soheil Mohajer and Shlomo Shamai

Lossy compression

- Lossless compression program: e.g. winzip, gzip, winrar, etc.
- Some signals are too costly to compress losslessly:
  - Audio: e.g. MP3
  - Images: e.g. JPEG, SPIHT and JPEG 2000
  - Video: e.g. MPEG2 (DVD), MPEG4 and h.264

Original image (929kB)

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Lossless compressed image (226kB)

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Lossy image (20.3kB)

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Lossier image (13.2kB)

Information theoretic lossy source coding

Shannon's lossy source coding problem (1948): the best we can do?

- Compress an i.i.d. source X with (long) block codes
- Distortion: measure the quality of the reconstruction

Rate-distortion function $R(D) = \min_{\mathbb{E}d(X,Y)\leq D} I(X;Y)$.

- Intuition on how to design optimal codes
- Benefit of knowing the best performance to expect

Lossy source coding with multiple users

How about for more than point-to-point case?

- Point-to-point case: rate and distortion
- General case: rate tuple and distortion tuple
  - Source coding: rate-distortion region
  - Joint source-channel coding: achievable-distortion region

Characterizing these regions is very difficult

- Only known for a few special cases
  - Wyner-Ziv coding, successive refinement and two-way communication
- Difficult even with restricted scope: e.g. Gaussian source only

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Approximate solutions

Can we find the solution approximately instead?

- Yes, “it’s easier to approximate"
- Particularly suitable for Gaussian problems

Good approximation is almost as useful as exact solution

- Intuition on how to design close-to-optimal codes.
- Benefit of knowing the optimal performance within some precision.

Bonus: may lead to precise solution in some special cases.

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What's in this talk?

A series of approximation results we recently presented

- Multiple descriptions
  - T., Mohajer, Diggavi, IT-09.
  - Mohajer, T., Diggavi, IT-10.
- Sending Gaussian source on broadcast channel
  - T., Diggavi, Shamai, IT-11.
- On the optimality of source-channel separation in networks
  - T., Chen, Diggavi, Shamai, Arxiv.

This talk:

- Summarize these results and discuss some related ones
- Connects some dots, and present several “new” results

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Outline

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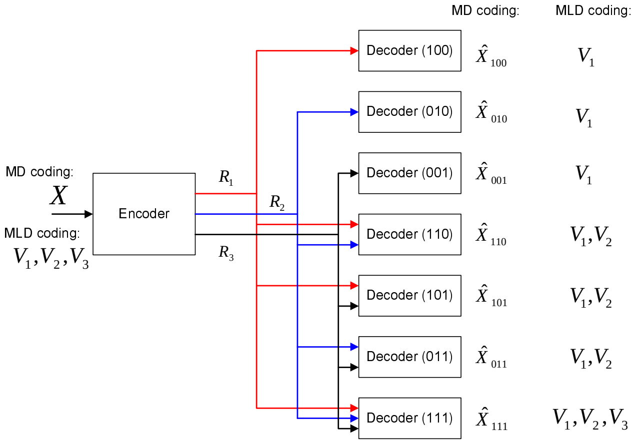
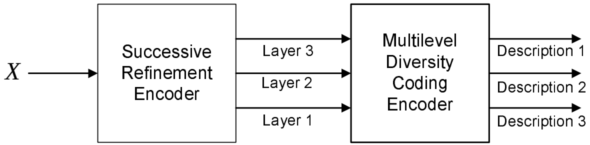
Gaussian multiple descriptions: symmetric distortions



- Lossy problem vs. lossless problem

"Approximating the Gaussian multiple description rate region under symmetric distortion constraints," IT-09

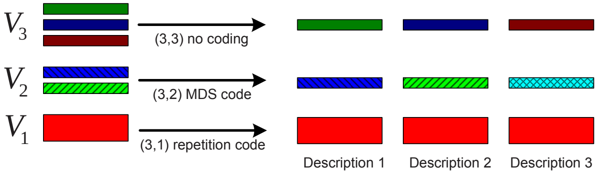
Gaussian multiple descriptions: symmetric distortions



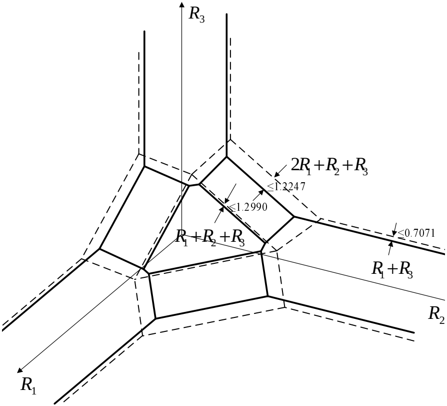
- Successive refinement: later pieces help refine earlier ones

What's the secret for MLD coding?

- Unequal loss protection

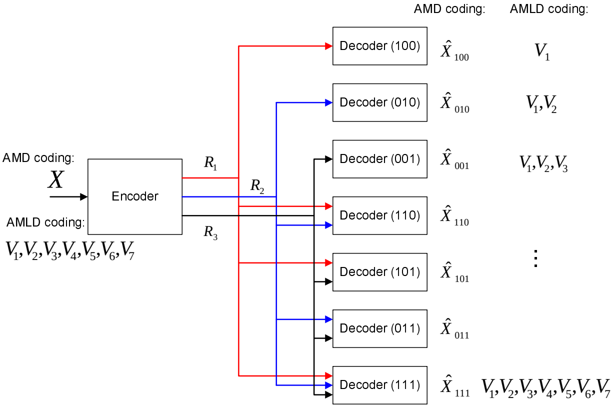


Rate region approximation



- Inner bound: multilevel diversity coding + successive refinement
- Gap can be reduced by using better coding scheme
- Generalized to more than three descriptions and other sources

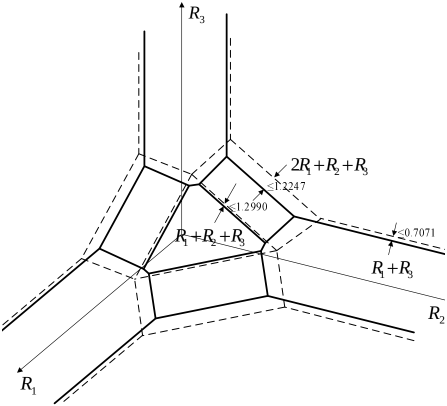
Multiple descriptions: asymm. distortions



- Lossless problem: asymmetric multilevel diversity coding

"Asymmetric multilevel diversity coding and asymmetric multiple descriptions," IT-10

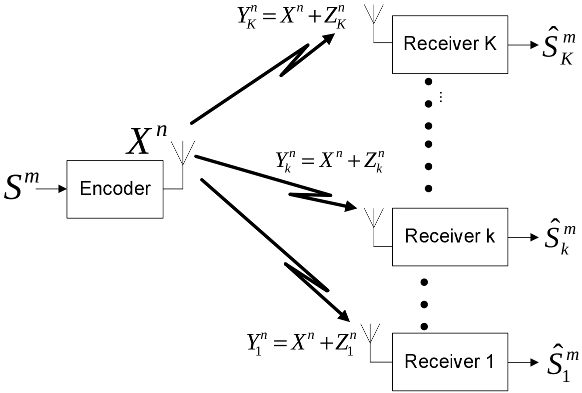
Rate region: asymm. multiple descriptions



- Asymm. multilevel diversity coding + successive refinement
- Note: asymm. MLD rate region is still open for > 3.

Outline

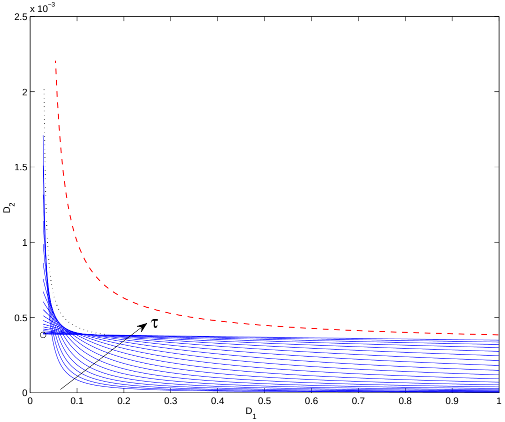
Gaussian source on Gaussian BC-channel



- Inner bound: separation-based scheme
  - Channel coding: degraded broadcast channel code
  - Source coding: successive refinement

"Approximate characterizations for the Gaussian source broadcast distortion region," IT-11.

Gaussian source on Gaussian BC-channel



- If the optimal scheme can achieve $(D_1, D_2, ..., D_K)$:
  - The separation scheme can achieve at least $(KD_1, KD_2, ..., KD_K)$.

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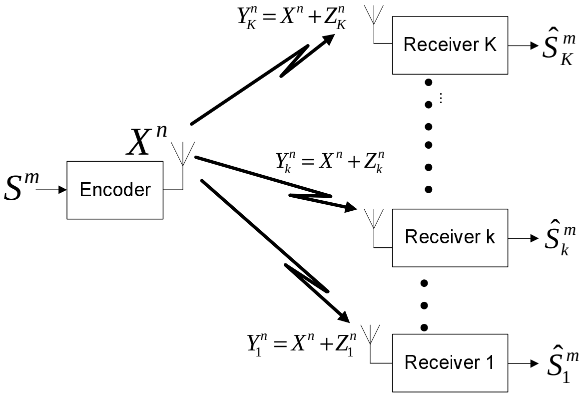
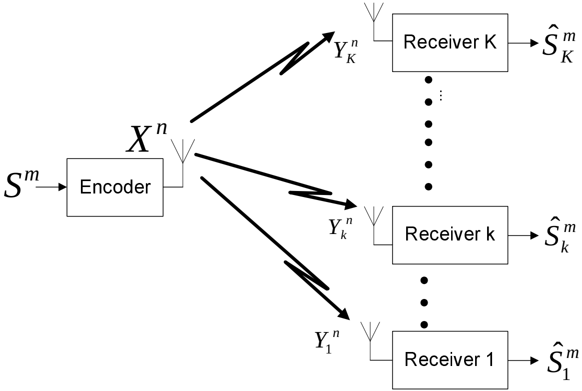
Gaussian source on General BC-channel



- Inner bound: separation-based scheme
  - Channel coding: broadcast with degraded message sets
- Same multiplicative gaps between inner and outer bounds

"Approximate characterizations for the Gaussian source broadcast distortion region," IT-11.

Gaussian source on General BC-channel



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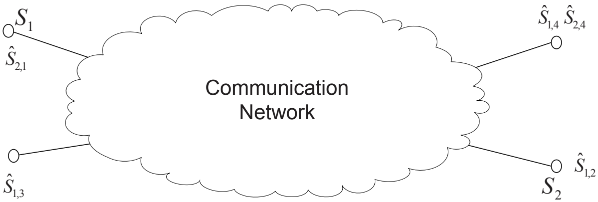
Outline

A network setting



Mutually independent sources and general multiuser channels

- A single source can be present at multiple nodes
- Each source is wanted at multiple sinks
- Restrictions on the distortion measure
  - Difference distortion measure for each single source: $f(s - \hat{s})$

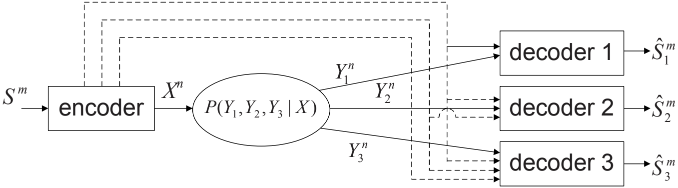
"Optimality and approximate optimality of source-channel separation in networks," Arxiv.

Approximate separation in network setting

Source-channel separation is approximately optimal:

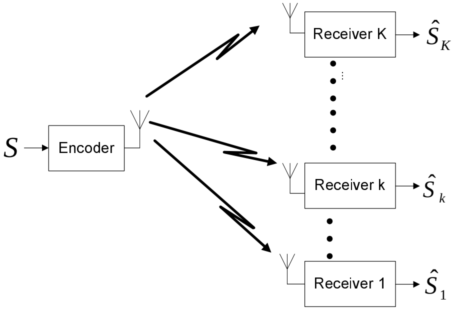
- Genie-provided links between source and destinations in a separation scheme:
  - As good as the optimal joint coding scheme

- The capacities of these genie links need not be too large



Outline

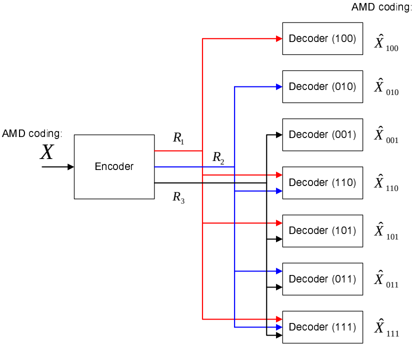
A hidden theme: Gaussian source broadcast

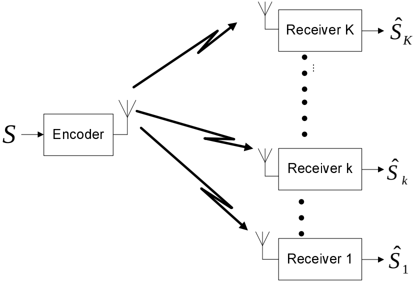


Gaussian sources on general broadcast channel: a general setting

- No restriction on the broadcast channels.
- Broadcast "channel" capacity may be unknown.
- Result: source-channel separation is approximately optimal.

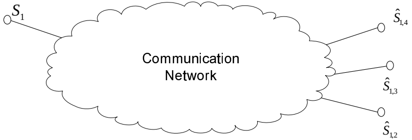
Gaussian MD and Gaussian source broadcast

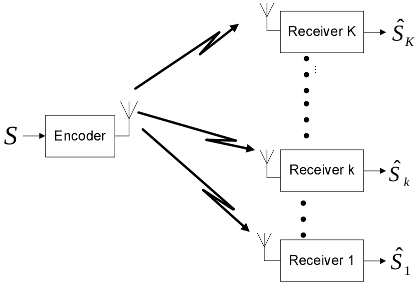




- Multiple descriptions: a deterministic broadcast channel
- Broadcast capacity = MLD rate region

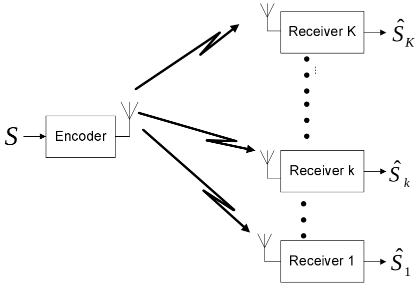
Gaussian source broadcast and network broadcast





- Network broadcast: an abstract broadcast channel
  - May have feedback and other multiuser channels in the network
- Interferences by multiple sources? Part of the channel code.

How did we get this general result?



- If certain distortions $D_1, D_2, ..., D_K$ is achievable
  - Fixing the enc/dec functions: induce a super channel

$$\mathbb{S}^m \to (\hat{\mathbb{S}}_1^m, \hat{\mathbb{S}}_2^m, ..., \hat{\mathbb{S}}_K^m)$$

  - This channel has some quality guarantees
  - Send the source S on this super-channel

More on the channel quality guarantee

Lemma (Channel qualities from distortions)

- *W: a random variable s.t. $\mathbb{E}d(S^m, g(W)) \leq D$.*
- *$U = S + V$ and $U' = S + V + V'$: V and V' indep. Gaussian random variables, with variance τ and $\tau' - \tau$.*

1. ***Mutual information bound***

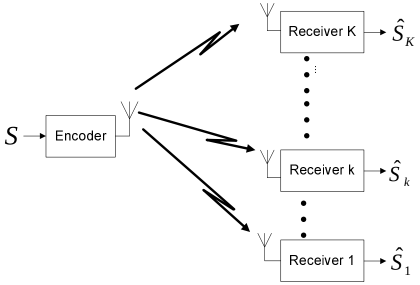
$$I(W; U'^m) \geq \frac{m}{2} \log \frac{1 + \tau'}{D + \tau'},$$

2. ***Bound on mutual information difference***

$$I(W; U^m) - I(W; U'^m) \geq \frac{m}{2} \log \frac{(1 + \tau)(D + \tau')}{(1 + \tau')(D + \tau)}.$$

Note: U and U' are r.v. additionally introduced.

More on the channel quality guarantee



On super-BC $\mathcal{S}^m \to (\hat{\mathcal{S}}_1^m, \hat{\mathcal{S}}_2^m, ..., \hat{\mathcal{S}}_K^m)$: each $\hat{S}_k^m$ is W.

- User-k in the super broadcast channel has rate

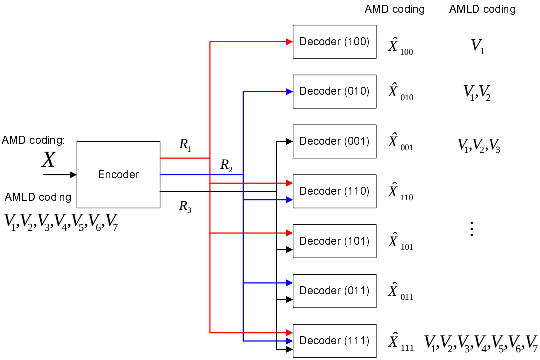
$$\frac{1}{2} \log \frac{(1+\tau_k) \prod_{j=2}^{k} (D_j + \tau_{j-1})}{\prod_{j=1}^{k} (D_j + \tau_j)}$$

- Using Gaussian R-D function $D = \exp(-2R)$
  - Separation scheme achieves some distortions (w/ parameters $\{\tau_j\}$)

First new case: revisiting asymm. Gaussian MD

- Asymm. MLD lossless problem difficult for $K > 3$
- Open: the asymm. MD approxmation problem for $K > 3$?

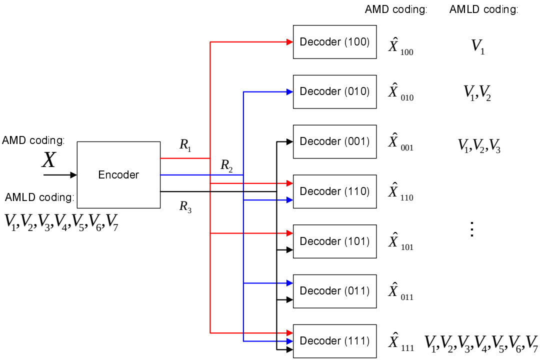
Question: is AMLD+successive refinement still approximately optimal?



First new case: revisiting asymm. Gaussian MD

Answer: yes! asymm. MLD is a deterministic broadcast channel

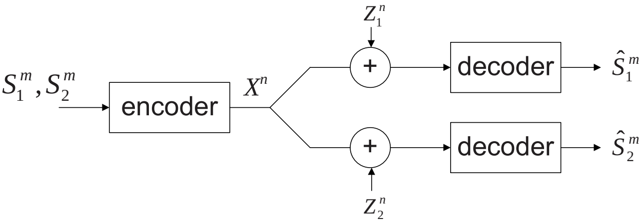
- If AMLD is solved ($K = 3$): approximate characterization
- If AMLD is open ($K > 3$): approximate optimality



Second new case: a less obvious problem

Broadcast correlated Gaussian sources on Gaussian channel

- Each user is interested in one source component
- Source bandwidth and channel bandwidth can be mismatched
- For bandwidth matched case, hybrid scheme is optimal (T., Diggavi, Shamai IT-11)

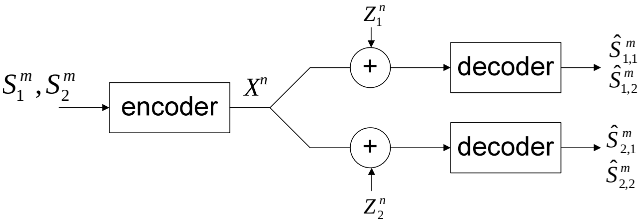


- Two sources are now dependent: can we still apply the result?

Second new case: a less obvious problem

An almost equivalent problem:

- Each user is interested in all components
- Replace the MSE distortion by covariance distortion
  - $\boldsymbol{D}_1$ at user 1, and $\boldsymbol{D}_2$ at user 2



- Separation scheme approximately optimal: genie link $\leq$ 2 bits

From Gaussian source to general sources

	Gaussian	General
Symmetric MD (IT-09)	Yes	Yes
Asymmetric MD (IT-10)	Yes	No
Source broadcast (IT-11)	Yes	No
Approx. separation (Arxiv)	Yes	Yes

- Approximation optimality of S-C separation with general sources
  - Sources under difference distortion measure $f(s - \hat{s})$
- Approximation results can all be extended to “general” sources.
- More general the model, looser the bound on the gap.

Forward coding schemes

	Separation	other
Symmetric MD (IT-09)	MLD+SR	PPR scheme
Asymmetric MD (IT-10)	AMLD+SR	
Source broadcast (IT-11)	Broadcast+SR	
Approx. separation (Arxiv)	Broadcast+SR	

Bound on the gap may be reduced with improved coding schemes.

Outer bounding proof techniques

- 1st: introduction of additional auxiliary random variables
- 2nd: the super channel abstraction

	1st	2nd
Symmetric MD (IT-09)	Yes	No
Asymmetric MD (IT-10)	Yes	No
Gaussian source/channel (IT-11)	Yes	No
Gaussian source on general channel (IT-11)	Yes	Yes
Approx. separation (Arxiv)	Yes	Yes

Type of approximations

- Rate type: rate-region between parallel hyperplanes
- Multiplicative type: multiplicative bound on distortion regions

Symmetric MD (IT-09)	Rate
Asymmetric MD (IT-10)	Rate
Gaussian source/channel (IT-11)	Multiplicative
Gaussian source on general channel (IT-11)	Multiplicative
Approx. separation (ISIT-10)	Rate

*Some cases the two types of approximations can be exchanged.

Type of approximations

a. c.: approximate characterization

- Approximate optimal architecture
- Characterization of the R-D (or distortion) region

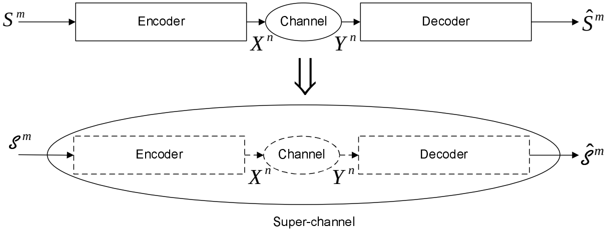
a. o.: approximate optimality

- Approximate optimal architecture

Symmetric MD (IT-09)	a. c.
Asymmetric MD $k = 3$ (IT-10)	a. c.
Asymmetric MD $k > 3$	a. o.
Gaussian source/channel (IT-11)	a. c.
Gaussian source on general channel (IT-11)	a. o.
Approx. separation (ISIT-10)	a. o.

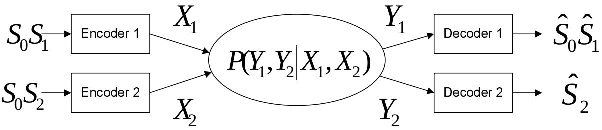
Outline

The super channel conversion



- Rate supported on the super channel $\geq I(S^m; \hat{S}^m) \geq mR(D)$
- Use this rate to encode source S? A separation-based scheme!
- Separation is optimal in point-to-point scenario (Shannon's)

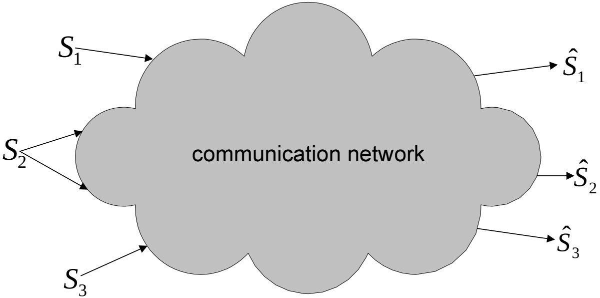
Independent Sources on Interference Channel



Source-channel separation is optimal

- Source and channel can have bandwidth mismatch.
- Channel feedbacks does not change the optimality.
- Result holds for lossless coding scenario.

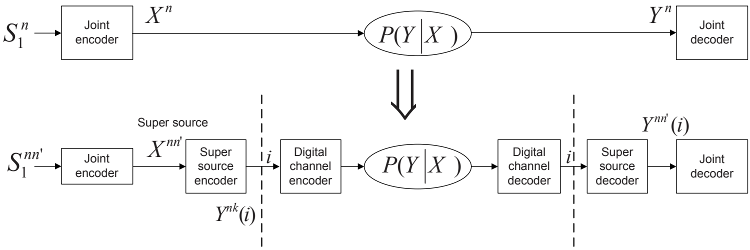
Generalization to networks



Network with independent sources, each source only has one sink

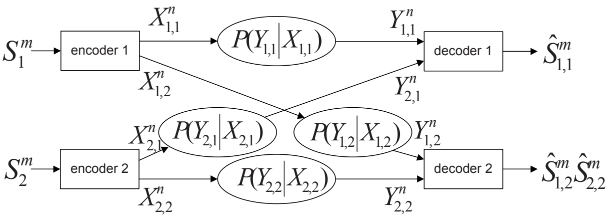
- A super interference channel.
- Source-channel separation is optimal!
- Interference by other inputs? Part of the super-channel code.

A dual result: the super source conversion



- View the channel output X^n as a super-source;
- Can encoder it using digital code because $I(X^n; Y^n) \leq nC$;
- A separation-based scheme!
- Separation is optimal in point-to-point scenario (Shannon's)

Dependent sources on orthogonal channels



Source-channel separation is optimal

- Source and channel can have bandwidth mismatch.
- Result holds for lossless coding scenario.

Generalization of the optimality to networks

- Network with orthogonal (line) channels;
- The extracted source coding problem needs interactive coding.

Outline

Concluding remarks

Multiple descriptions $\rightarrow$ Gaussian source broadcast $\rightarrow$ approximate optimality of S-C separation in network

- Multiple descriptions: source broadcast on deterministic channel
- Network source broadcast: source broadcast on abstract channels

Approximate characterization vs. approximate optimality

- Even if a. c. is not available, a. o. gives architecture insight

Concluding remarks

Multiple descriptions $\rightarrow$ Gaussian source broadcast $\rightarrow$ approximate optimality of S-C separation in network

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Approximate characterization vs. approximate optimality

- Even if a. c. is not available, a. o. gives architecture insight

Some open problems

P1: Gaussian source coding on general broadcast channels

- Common messages and private messages.
- May help the approximation?
- The lossless problem: 3 receivers and 7 descriptions (each received by a receiver subset)

P2: The missing approximate optimality result?

- Separation is optimal for
  - Unicast of independent sources on a general channel network;
  - Dependent sources on an orthogonal channel network.
- Separation is approximately optimal for
  - Multicast of independent sources on a general channel network;
  - Dependent sources on a "partially orthogonal" channel network?