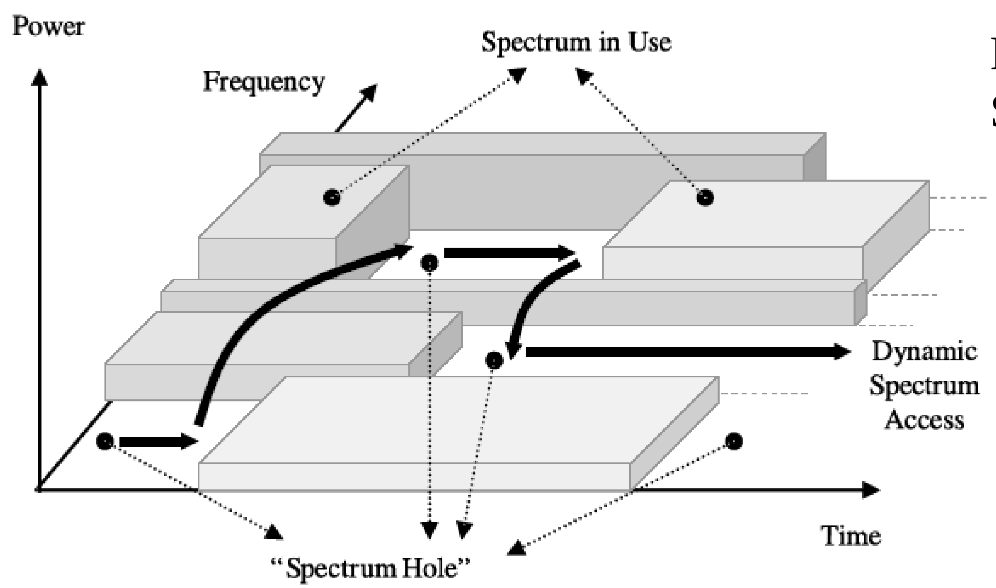
Sensing and Recognition When Primary User has Multiple Transmit Powers

Feifei Gao

Institute of Information Processing
Department of Automation
Tsinghua University

Cognitive Radio

- Overview
  - Interleaved: Spectrum sensing based.



Primary User
Secondary User

  - Underlay: Interference temperature

Spectrum Sensing

- Spectrum Sensing
  - Matched Filter
  - Cyclostationary
  - Energy Detection → Most popular
- Accompanied Research:
  - Parameter Uncertainty
  - Cooperative Sensing
  - Secondary Games
  - Sensing Throughput Trade-off
  - Imperfect Sensing
  - Combined with Multi-Antenna, OFDM, Relay, Secrecy….
- One critical "Bug" exists:
  - Assume PU has only ONE power level!!!

Many Others

- Most Standards says:
  - PU will work on different power levels depending on the rate, bandwidth, environment.
  - For example, in IEEE 802.11, GSM, LTE, etc.
- If SU knows PU’s current power (each time), traditional method works. But….
- Other supports for studying the varying power levels
  - We spend so many effort in designing the power allocation.
  - Theoretical interest towards more “cognition”
- A more reasonable scenario is:
  - SU knows all the power levels of PU but it does not know which level PU currently stays.

- Spectrum sensing with multiple PU power levels:
  - Primary Target: Detect the presence of PU
  - Secondary Target: Find the status of PU
- Benefit?
  - More Information (nothing bad to know more)
  - Further Strategies, example
  - Any other you can imagine?
- A possibly new (small) direction in CR?
  - Some new issues deserve (re)-investigation

清华大学
Tsinghua University

Part I
Spectrum Sensing

System Model

- N Power level $P_{i+1} > P_i > 0$

$$x_l = \begin{cases} n_l & \mathcal{H}_0 \\ \sqrt{P_i}\sqrt{g}s_l + n_l & \mathcal{H}_i, \; i = 1, 2, ..., N \end{cases}$$

  with $\sum_{i=0}^{N} P(\mathcal{H}_i) = 1$

- It can be proved that energy detection is optimal under Gaussian signal/noise
- Energy form M received symbols $y = \sum_{l=1}^{M} |x_l|^2$

$$p(y|\mathcal{H}_i) = \frac{y^{\frac{M}{2}-1} e^{-\frac{y}{2\sigma^2 + 2gP_i}}}{\Gamma(\frac{M}{2})(2\sigma^2 + 2gP_i)^{\frac{M}{2}}}$$

清华大学 Tsinghua University

Spectrum Sensing: Approach I

- "Presence" first, "Status" second
- The presence of PU $\mathcal{H}_{\text{on}}$ with $\Pr(\mathcal{H}_{\text{on}}) = \sum_{i=1}^{N} \Pr(\mathcal{H}_i)$. Then

$$p(y|\mathcal{H}_{\text{on}}) = \frac{1}{\Pr(\mathcal{H}_{\text{on}})} \sum_{i=1}^{N} p(y|\mathcal{H}_i)\Pr(\mathcal{H}_i)$$

- Detection rule

$$p(\mathcal{H}_{\text{on}}|y) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_{\text{on}}}{\gtrless}} p(\mathcal{H}_0|y)$$

which is simplified to

$$\sum_{i=1}^{N} p(y|\mathcal{H}_i)\Pr(\mathcal{H}_i) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_{\text{on}}}{\gtrless}} p(y|\mathcal{H}_0)\Pr(\mathcal{H}_0)$$

No closed form expression

- Detect the Status of PU if “on”

$$p(\mathcal{H}_i|y,\mathcal{H}_{\text{on}}) \underset{j}{\overset{i}{\gtrless}} p(\mathcal{H}_j|y,\mathcal{H}_{\text{on}}),\ \ \forall i,j \geq 1$$

- With $p(\mathcal{H}_{\text{on}}|\mathcal{H}_i,y)=1$ and Bayes Rule, there is

$$p(y|\mathcal{H}_i)\text{Pr}(\mathcal{H}_i) \underset{j}{\overset{i}{\gtrless}} p(y|\mathcal{H}_j)\text{Pr}(\mathcal{H}_j),\ \ \forall i,j \geq 1$$

- The final decision rule can be derived as

$$\mathcal{R}(\mathcal{H}_i) = \left\{ y \middle| \max_{j<i} \Theta(i,j) < y < \min_{j>i} \Theta(i,j),\ \ \forall i \geq 1 \right\}$$

See expression of $\Theta(i,j)$ next page

$$\Theta(i,j) = \frac{2(\sigma^2 + gP_i)(\sigma^2 + gP_j)}{g(P_i - P_j)} \ln\left[\left(\frac{\sigma^2 + gP_i}{\sigma^2 + gP_j}\right)^{\frac{M}{2}} \frac{\Pr(\mathcal{H}_j)}{\Pr(\mathcal{H}_i)}\right]$$

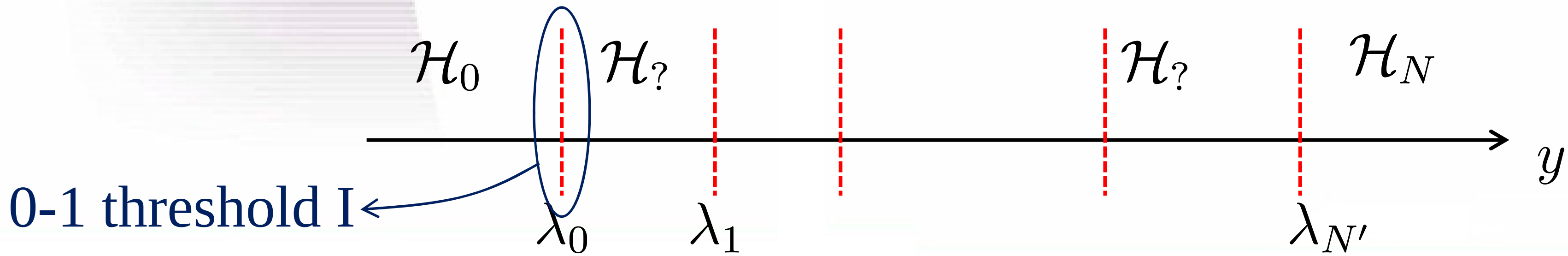
- Interesting phenomenon, when

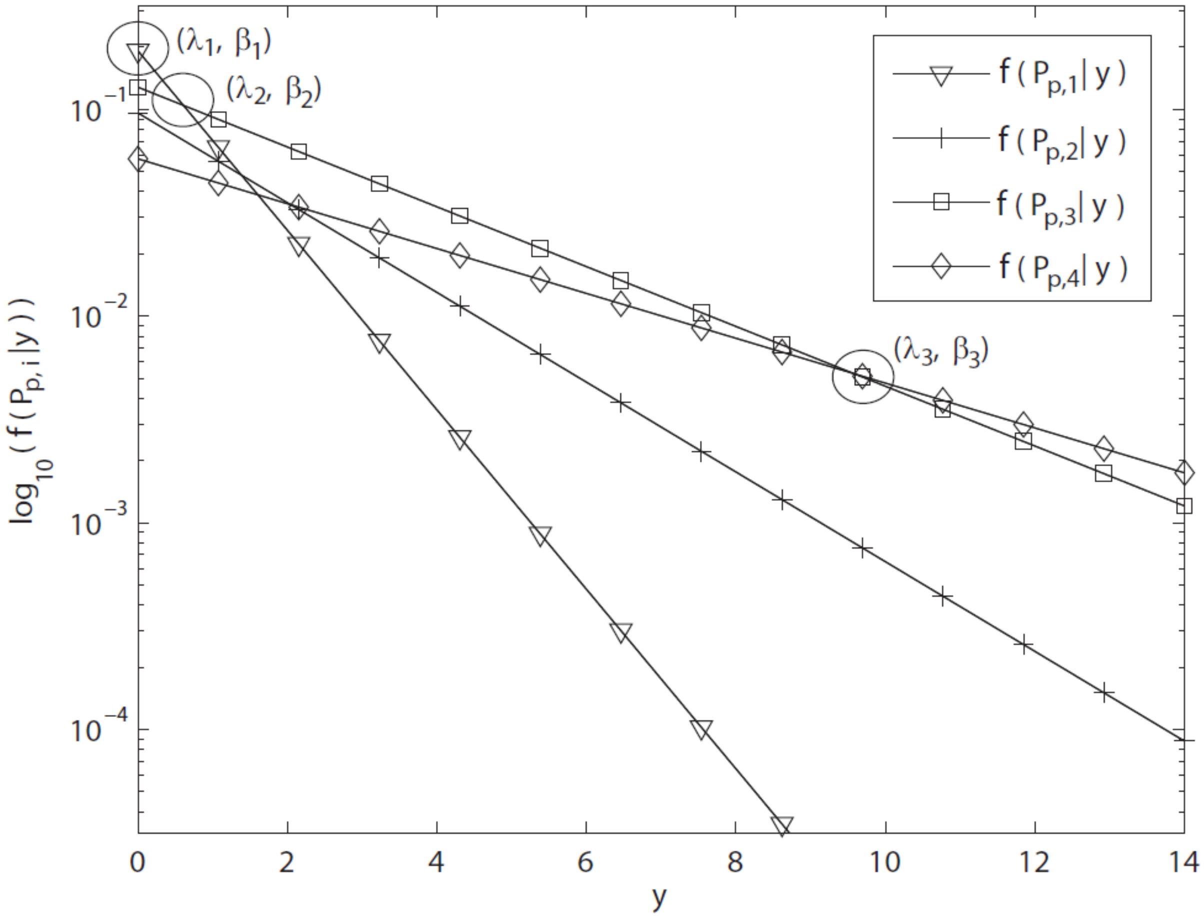
$$\max_{j<i} \Theta(i,j) > \min_{j>i} \Theta(i,j)$$

Hiding Status Effect

- It can be proved that if $p(\mathcal{H}_i) = p(\mathcal{H}_j),\ \forall i,j \geq 1$

$$\mathcal{R}(\mathcal{H}_i) = \left\{ y \middle| \Theta(i,i-1) < y < \Theta(i,i+1),\ \forall i \geq 1 \right\}$$




(λ_1, β_1)
(λ_2, β_2)
(λ_3, β_3)
$f(P_{p,1}|y)$
$f(P_{p,2}|y)$
$f(P_{p,3}|y)$
$f(P_{p,4}|y)$
$\log_{10}(f(P_{p,i}|y))$
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0
2
4
6
8
10
12
14
y

- Important result: for those detectable j

$$\Pr(\mathcal{H}_j|\mathcal{H}_i) = \frac{\gamma(\frac{M}{2}, \frac{\lambda_j}{2N_0+2P_{p,i}\gamma_1})}{\Gamma(\frac{M}{2})} - \frac{\gamma(\frac{M}{2}, \frac{\lambda_{j+1}}{2N_0+2P_{p,i}\gamma_1})}{\Gamma(\frac{M}{2})}$$

  others zero.

- Discussion
  - False alarm

$$\mathrm{P}_f = \Pr(\mathcal{H}_{\mathrm{on}}|\mathcal{H}_0)$$

  - Detection Probability:

$$\mathrm{P}_d = \Pr(\mathcal{H}_{\mathrm{on}}|\mathcal{H}_{\mathrm{on}}) = \sum_{i=1}^{N} \Pr(\mathcal{H}_i|\mathcal{H}_i)\Pr(\mathcal{H}_i)$$

  - New Metrics $\Pr(\mathcal{H}_j|\mathcal{H}_i)$

Spectrum Sensing: Approach II

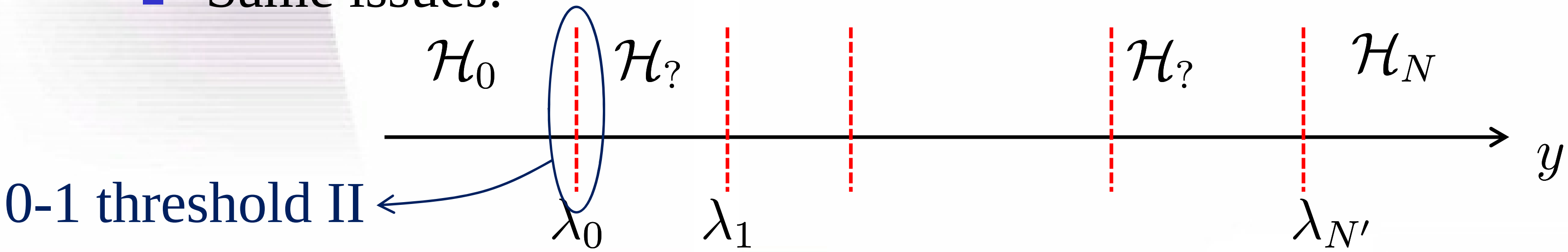
- Detect the status directly:

$$p(\mathcal{H}_i|y) \underset{j}{\overset{i}{\gtrless}} p(\mathcal{H}_j|y), \ \forall i, j$$

- From previous:

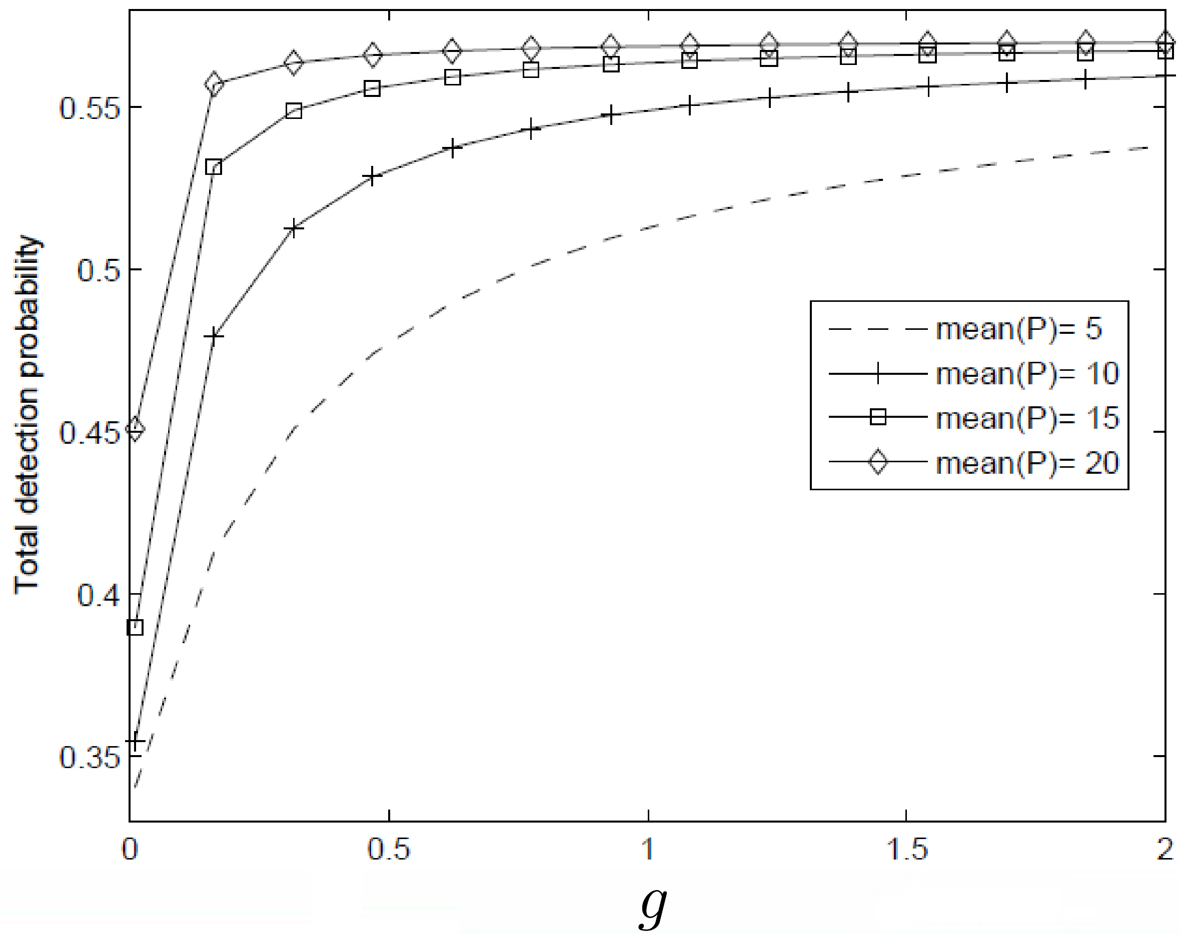
$$\mathcal{R}(\mathcal{H}_i) = \left\{ y \middle| \max_{j<i} \Theta(i,j) < y < \min_{j>i} \Theta(i,j), \ \forall i \right\}$$

- Same issues:



- Interesting Discussions:
  - Thresholds from two different approach the same?
  - Neyman Pearson Criterion applicable?
  - Definition of detection probability redefined?
  - …………………………….

Simulation



Total detection probability
0.35
0.4
0.45
0.5
0.55
0
0.5
1
1.5
2
g
mean(P)= 5
mean(P)= 10
mean(P)= 15
mean(P)= 20

Part II
Cooperative Sensing

- Cooperative sensing is used for combat both the fading and the noise effect.
- Existing cooperative schemes (decision fusion):
  - AND
  - OR
  - k out of K

Applicable in multiple power-level? **NO!!!**

- Need to develop new rules here
  - Hard-fusion (majority)
  - Soft-fusion (posterior probability)

Majority Fusion

- Define the decision vector

$$\vec{d} = \{d_0, \ldots, d_N\}$$

with $\sum_{j=0}^{N} d_j = K$

  - Total number of possible $\vec{d}$ is $(N+1)^K$

- Majority rule

$$\hat{j} = \max_{j} \ d_j$$

Be careful about the simultaneous maximum

- The decision probability

$$\Pr_m(\mathcal{H}_j|\mathcal{H}_i) = \sum_{\vec{d} \in \mathcal{S}_{m_j}} \Pr(\vec{d}\,|\mathcal{H}_i)$$

with

$$\mathcal{S}_{m_j} = \left\{ \vec{d} \,\middle|\, d_j = \max\{d_0, d_1, \ldots, d_N\} \right\}$$

- Further assumption:
  - Existing work focus on the same fading scenario (reason?)
  - With different fading, the theoretical derivation is tedious
- Then

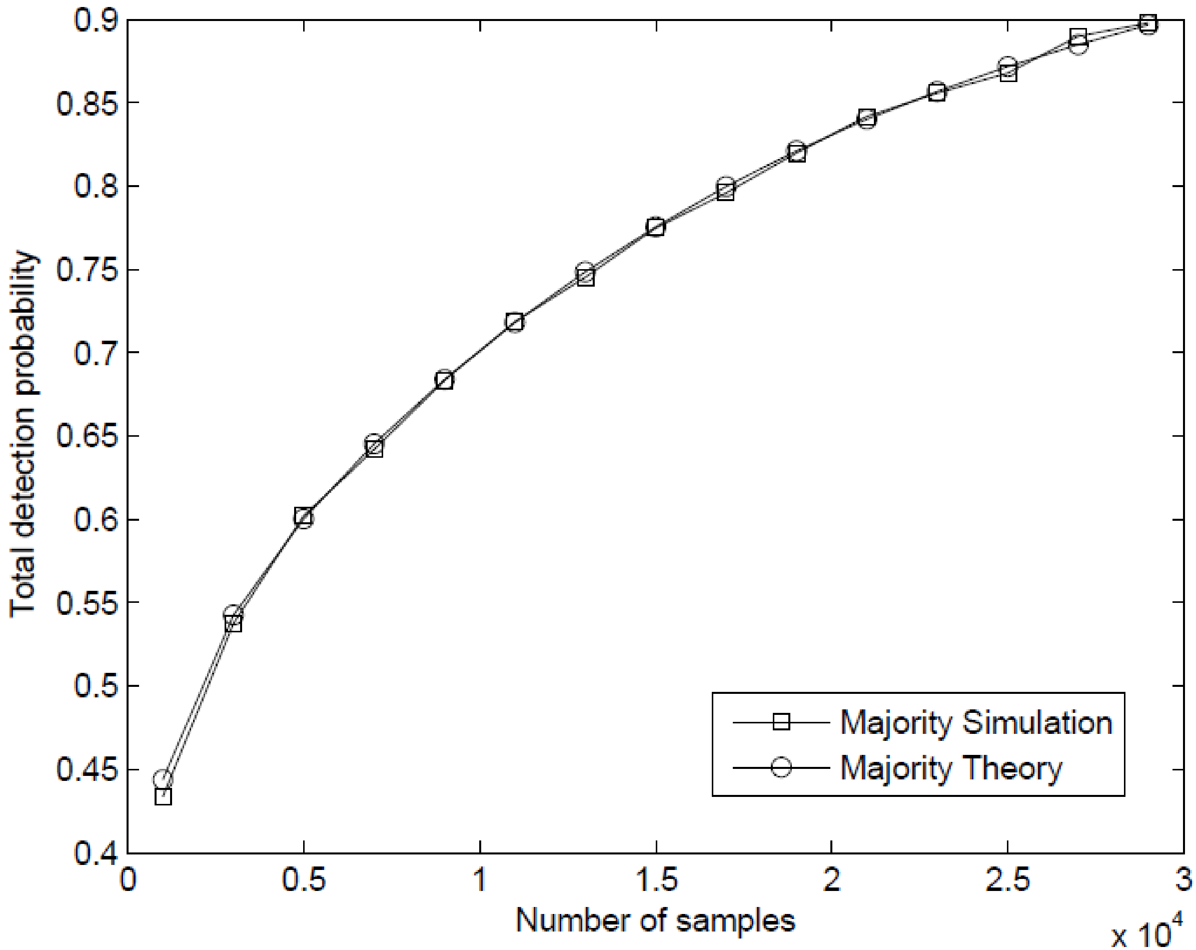
$$\Pr(\vec{d}|\mathcal{H}_i) = \frac{K!}{\prod_{l=0}^{N} d_l!} \prod_{n=0}^{N} \Pr(\mathcal{H}_n|\mathcal{H}_i)^{d_n}$$

and closed-form $\Pr_m(\mathcal{H}_j|\mathcal{H}_i)$ can be derived (very complicated)

The only analytical result for majority law seen so far

- Check total detection probability

$$\mathrm{P_d} = \frac{1}{\sum_{n=1}^{N} \Pr(\mathcal{H}_n)} \sum_{i=1}^{N} \Pr_m(\mathcal{H}_i|\mathcal{H}_i)\Pr(\mathcal{H}_i)$$


Total detection probability
0.9
0.85
0.8
0.75
0.7
0.65
0.6
0.55
0.5
0.45
0.4
0
0.5
1
1.5
2
2.5
3
Number of samples
x 10^4
Majority Simulation
Majority Theory

Optimal Fusion

- Majority decision does not consider the prior information of each $\mathcal{H}_i$
  - For example: $d_i < d_j$ but $\Pr(\mathcal{H}_i) \gg \Pr(\mathcal{H}_j)$
  - Need the information of $\Pr(\mathcal{H}_i)$ at fusion center.
- Optimal Fusion

$$\hat{j} = \arg\max_j \Pr\left(\mathcal{H}_j|\vec{d}\,\right) = \arg\max_j \Pr\left(\vec{d}\,|\mathcal{H}_j\right)\Pr(\mathcal{H}_j).$$

- The decision probability is

$$\Pr_o(\mathcal{H}_j|\mathcal{H}_i) = \sum_{\vec{d}\in\mathcal{S}_o} P(\vec{d}\,|\mathcal{H}_i) \qquad (*)$$

with

$$\mathcal{S}_{o_j} = \{\vec{d}\,|\text{ those } \vec{d} \text{ that make } \hat{j} = j \text{ in } (*)\}$$

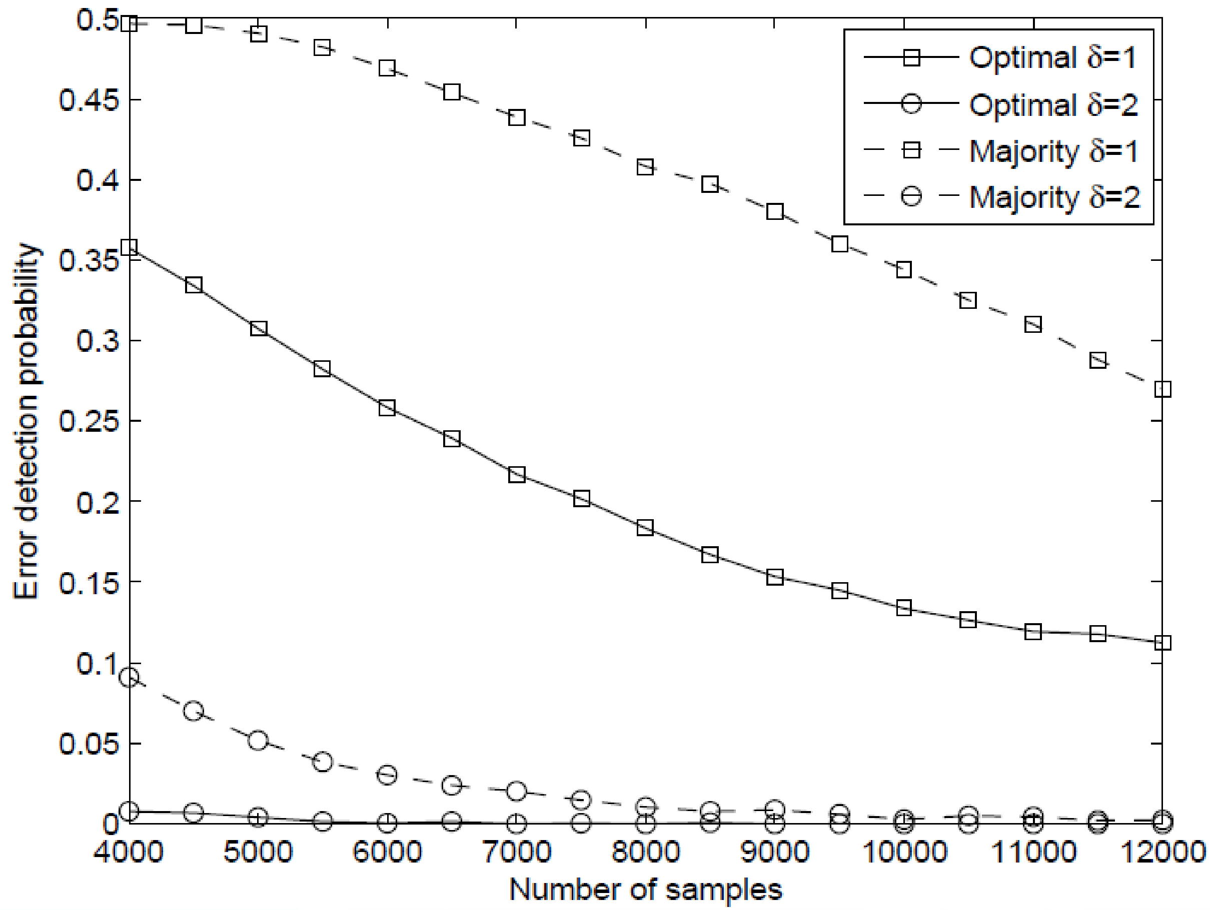
- One step further if we assume the same fading again…

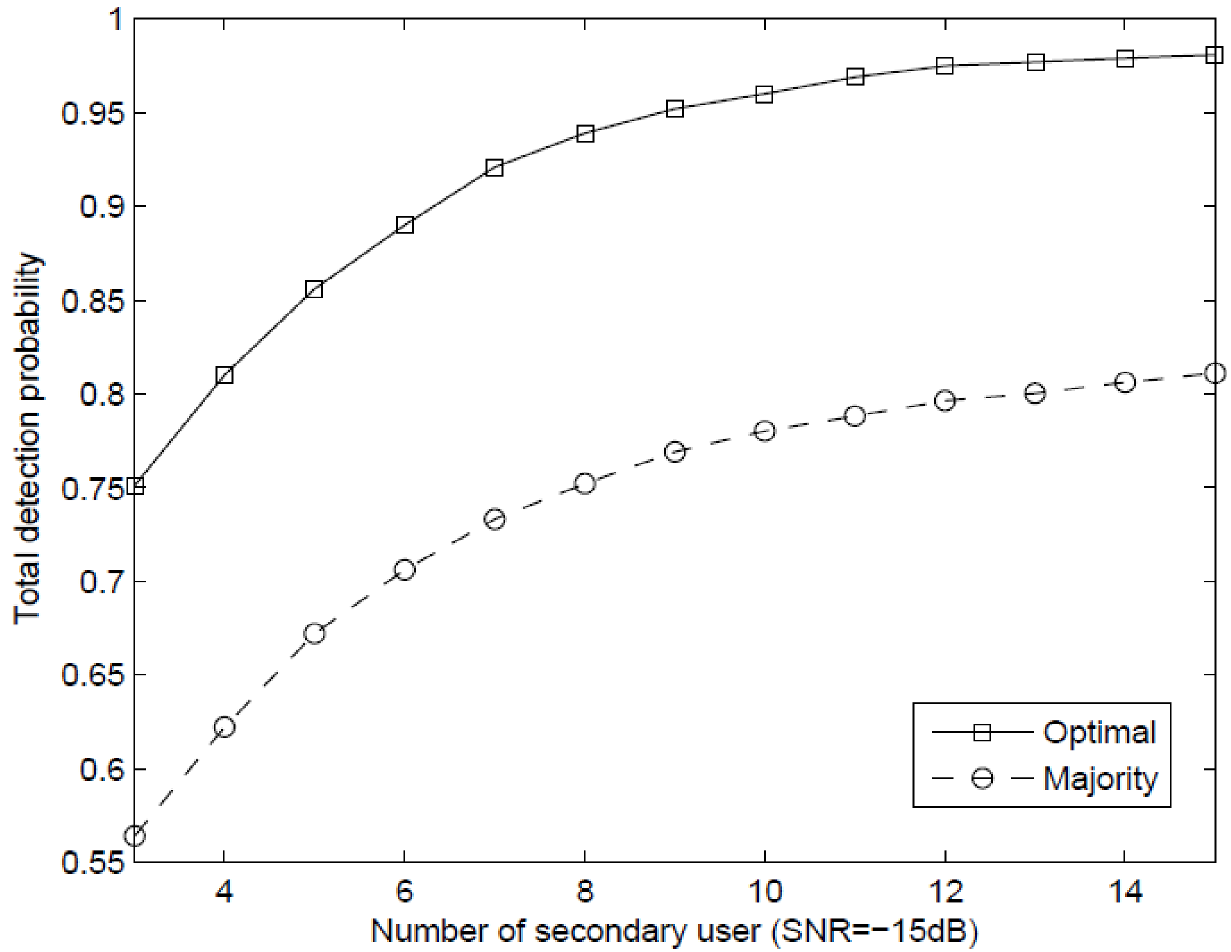
$$\hat{j} = \arg\max_{j} \; \Pr(\mathcal{H}_j) \prod_{n=0}^{N} \Pr(\mathcal{H}_n | \mathcal{H}_j)^{d_n}$$

$$= \arg\max_{j} \; \log \Pr(\mathcal{H}_j) + \sum_{n=0}^{N} d_n \log \Pr(\mathcal{H}_n | \mathcal{H}_j)$$

No closed form solution for $\mathcal{S}_{o_j}$, but easy offline computation

Simulations




Total detection probability
Number of secondary user (SNR=−15dB)
Optimal
Majority
1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0.6
0.55
4
6
8
10
12
14

Part III
Imperfect Parameters

On Going

- Unknown noise variance
  - Unbounded (SNR Wall effect)
  - Bounded
- Unknown channel
  - Not possible unless bounded
  - Statistics being known
- Unknown power level
  - The number of power level is known
  - The number of power level is unknown

  } **Classification**
  More Cognition
- Many others….

Conclusions

- What we have done in CR:
  - We considered a more practical scenario
  - We designed the optimal detection algorithm
  - We analytically characterize the performance
  - Cooperative sensing looks to be very different
  - Imperfect parameters seems to have some differences
- Future?
  - Some new phenomenon need to be studied.
  - Some old topics in CR deserve re-investigation

Thank you!!!