MIMO Two-Way Relaying: A Space-Division Approach

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Outline

- Background and Motivation
- MIMO TWRC Model and Previously
- Reduced-Dimension Precoding
- Space-Division Approach
- Conclusions and Future Work

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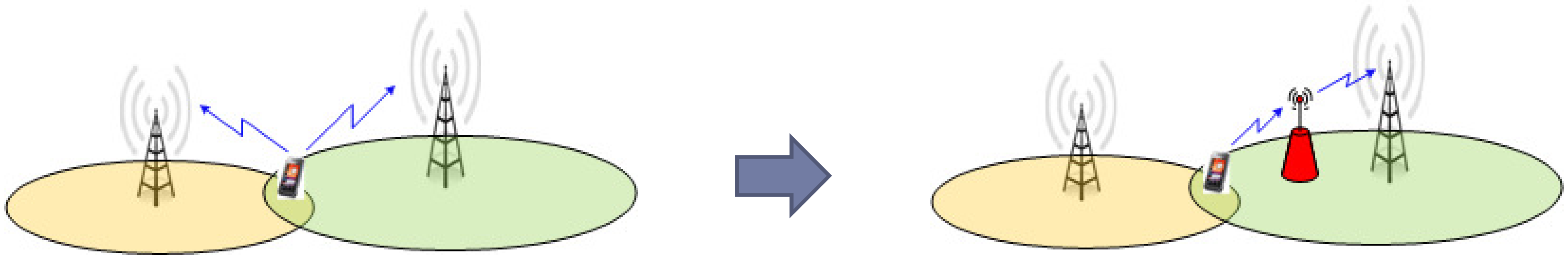
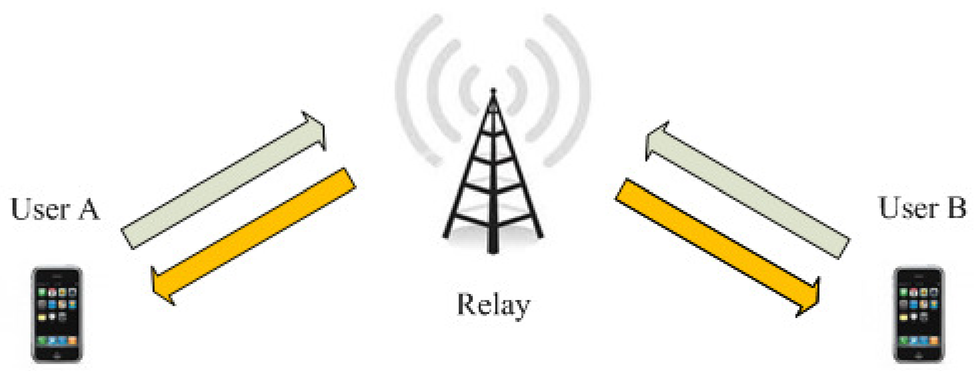
Cellular Networks: Needs for Relays

- Drivers for Future Cellular Systems
  - To achieve high data rates
- Advanced Technologies
  - MIMO, OFDM, advanced error-correction coding
- Needs for Relays
  - To improve data transmission at the cell edge



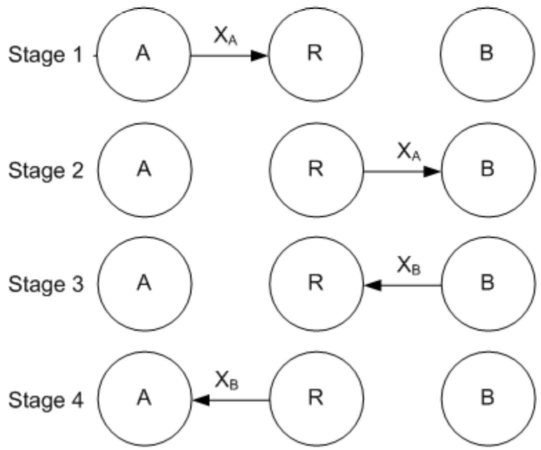
Network Coding for Relay Channels

- Physical-layer Network Coding (PNC)
  - An emerging technique for efficient transmission over relay networks
- Two-Way Relay Channel (TWRC)
  - Users A and B exchange information via a relay R

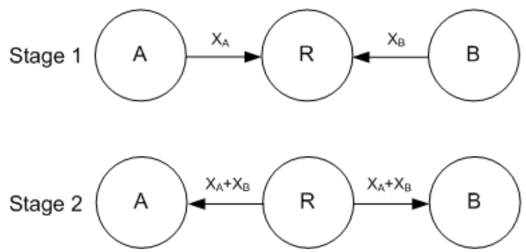


Conventional Routing vs. PNC

- Conventional 4-stage Routing
- Physical-layer Network-Coding (PNC)
  - Reduce 4 stages to 2 stages
  - Potentially 100% throughput burst



Conventional 4-stage Routing



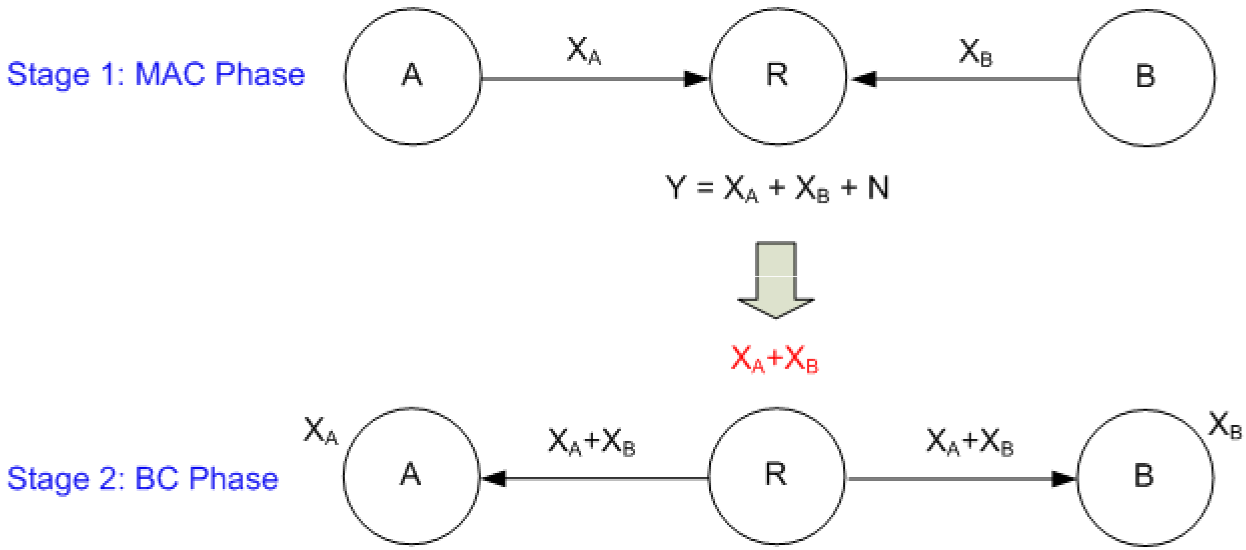
2-stage PNC

Big Challenge for PNC

How to incorporate the advanced technologies (including error-correction coding and MIMO) into PNC networks, and push them towards their limits?

Two-Way Relaying Protocol

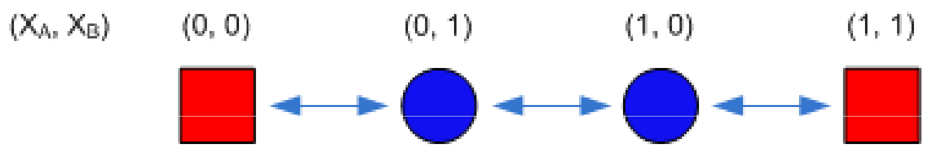
- Two-way transmission over TWRC



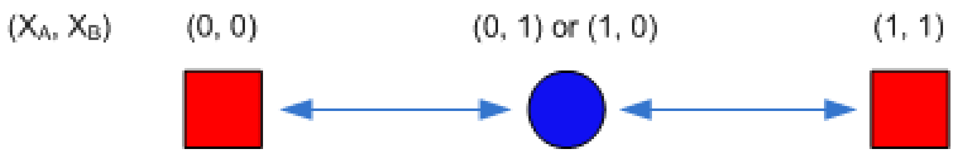
- The difficulty is how to detect/decode X_A+X_B from Y.
- We mostly focus on the MAC phase.

Complete-Decoding vs. PNC Decoding

- Assume BPSK for both users.
- Received signal at the relay: $Y = X_A + X_B + N$
- Complete decoding: to decode both X_A and X_B
  - Needs to distinguish 4 different constellation points



- PNC decoding: to decode $X_A + X_B$
  - Only need to distinguish red or blue
  - Merge the constellation points at the relay using power control



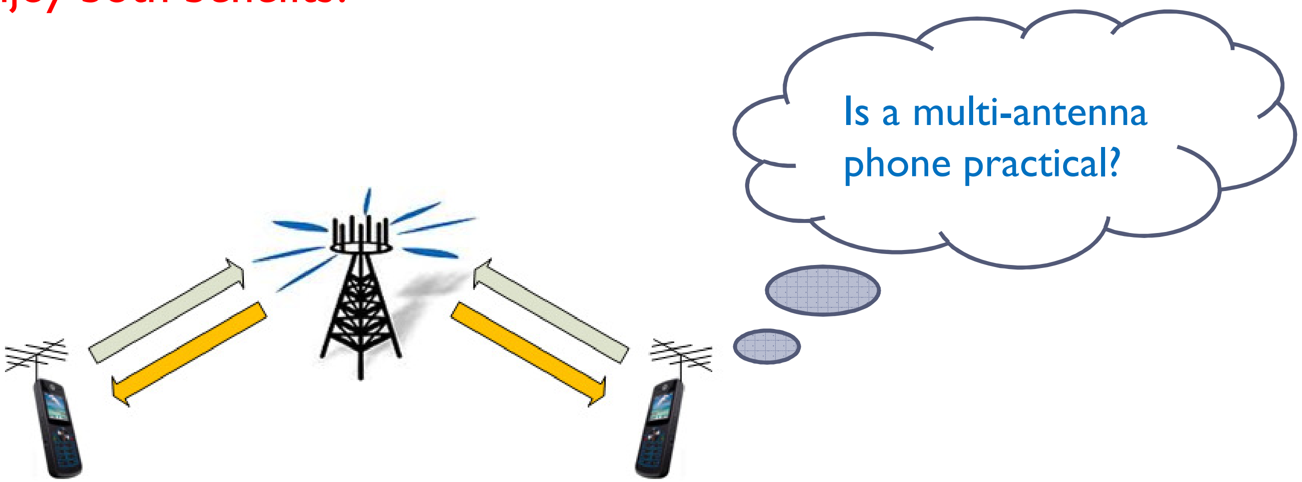
PNC with Error-Correction Coding

- Difficulties to incorporate channel coding
  - Channel coding involves multiple channel uses.
  - The decoding codebook C seen at the relay: $X_A^n + X_B^n$
  - How to align signal at the codeword level such that the size of C is minimized?

- Nested Lattice Coding for Gaussian TWRC
  - Use nested lattice codes to merge the signal constellation points
  - Asymptotically achieve capacity at high SNR

When MIMO meets PNC…

- MIMO Two-Way Relaying
  - PNC: 4-phase to 2-phase, doubling throughput
  - Multiple-Input Multiple-Output (MIMO): multifold throughput boost
  - Can we enjoy both benefits?



MIMO Two-Way Relay Channel

Future of Apple and Samsung

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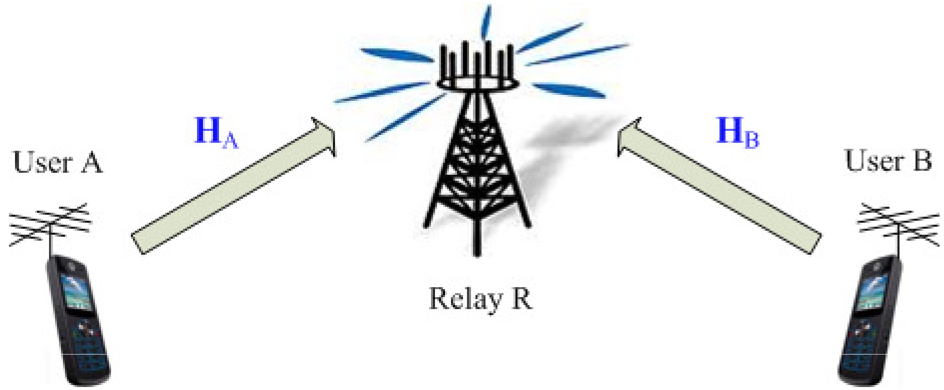
- Conclusions and Future Work

MIMO Two-Way Relay Channel



- Users A and B each equipped with n_T antennas and relay with n_R antennas
- MAC phase: $\mathbf{y} = \mathbf{H}_A \mathbf{x}_A + \mathbf{H}_B \mathbf{x}_B + \mathbf{n}$
- Relay function: $\mathbf{x}_R = f(\mathbf{y})$
- BC phase: $\mathbf{y}_A = \mathbf{G}_A \mathbf{x}_R + \mathbf{n}_A$ and $\mathbf{y}_B = \mathbf{G}_B \mathbf{x}_R + \mathbf{n}_B$
- Our goal is to design an efficient transceiver system.

Previously: Zero-Forcing (ZF) Precoding

- Assumption: The number of user antennas (n_T) is no less than that of the relay antennas (n_R), i.e., $n_T >= n_R$.

- MAC phase model: $\mathbf{y} = \mathbf{H}_A \mathbf{x}_A + \mathbf{H}_B \mathbf{x}_B + \mathbf{n}$
- Precoder at User A: $\mathbf{x}_A = \mathbf{H}_A^{-1} \mathbf{\Psi}_A^{1/2} \mathbf{c}_A$
- Precoder at User B: $\mathbf{x}_B = \mathbf{H}_B^{-1} \mathbf{\Psi}_B^{1/2} \mathbf{c}_B$

- $\mathbf{\Psi}_A$ and $\mathbf{\Psi}_B$ are diagonal matrices for power allocation
- $\mathbf{c}_A$ and $\mathbf{c}_B$ contain iid coded streams
- Equivalent parallel channels:

$$\mathbf{y} = \mathbf{\Psi}_A^{1/2} \mathbf{c}_A + \mathbf{\Psi}_B^{1/2} \mathbf{c}_B + \mathbf{n}$$

- Performance loss is significant due to channel inverse.

Previously: A Remedy for Zero-Forcing

- MAC phase model: $\mathbf{y} = \mathbf{H}_A \mathbf{x}_A + \mathbf{H}_B \mathbf{x}_B + \mathbf{n}$
- Rotated by a unitary matrix $\mathbf{K}$ at the relay:

$$\mathbf{K}\mathbf{y} = \mathbf{K}\mathbf{H}_A \mathbf{x}_A + \mathbf{K}\mathbf{H}_B \mathbf{x}_B + \mathbf{K}\mathbf{n}$$

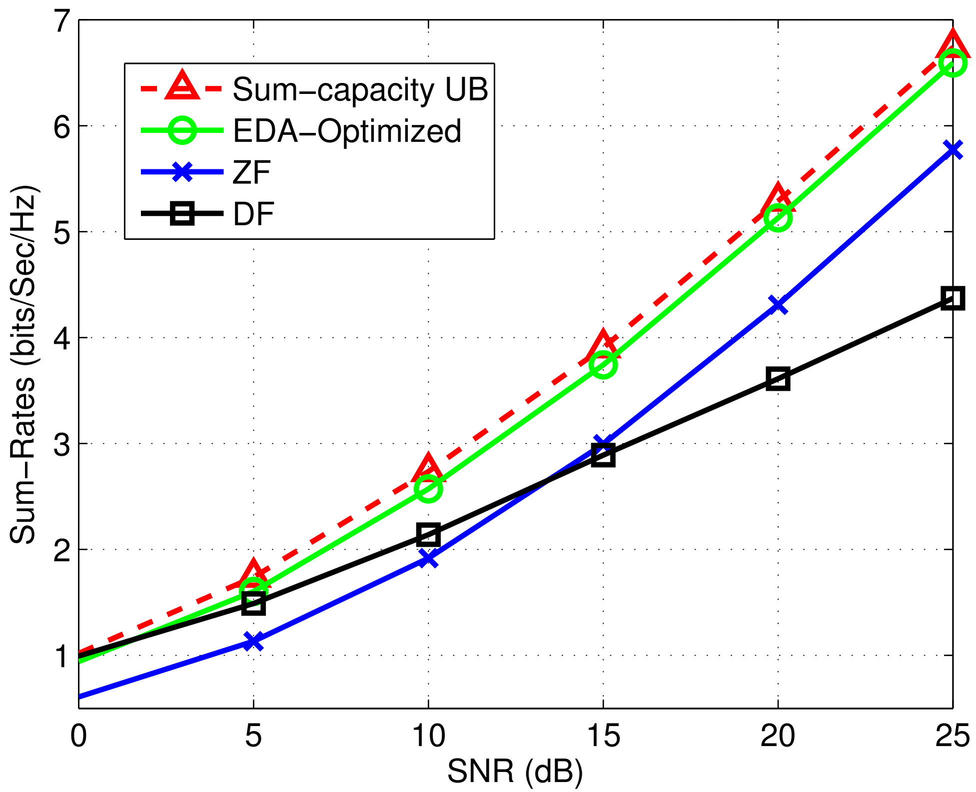
- Precoder at User A: $\mathbf{x}_A = (\mathbf{K}\mathbf{H}_A)^{-1} \boldsymbol{\Psi}_A^{1/2} \mathbf{c}_A$
- Precoder at User B: $\mathbf{x}_B = (\mathbf{K}\mathbf{H}_B)^{-1} \boldsymbol{\Psi}_B^{1/2} \mathbf{c}_B$
- $\boldsymbol{\Psi}_A$ and $\boldsymbol{\Psi}_B$ are diagonal matrices for power allocation.
- $\mathbf{c}_A$ and $\mathbf{c}_B$ contain iid coded streams with unit power.

- Equivalent parallel channels:

$$\mathbf{K}\mathbf{y} = \boldsymbol{\Psi}_A^{1/2} \mathbf{c}_A + \boldsymbol{\Psi}_B^{1/2} \mathbf{c}_B + \mathbf{K}\mathbf{n}$$

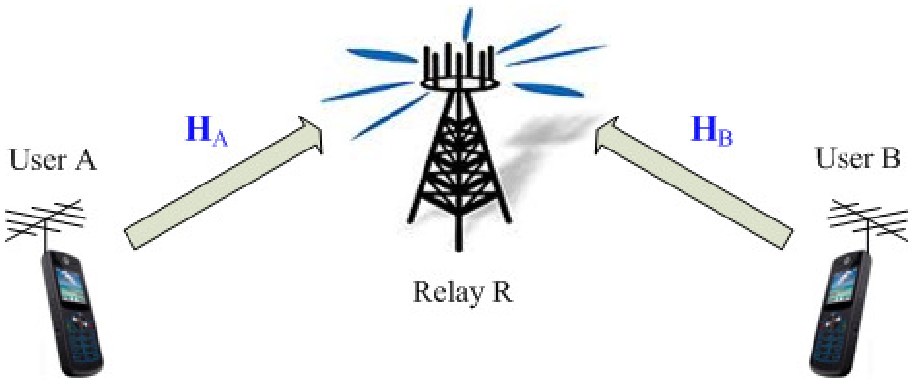
- With a proper choice of $\{\mathbf{K}, \boldsymbol{\Psi}_A, \boldsymbol{\Psi}_B\}$, the power loss problem can be significantly mitigated.

Previously: Average achievable sum-rates of various schemes for Rayleigh fading MIMO TWRC with $n_T = 2$ and $n_R = 2$



What if $n_T < n_R$?

Zero-Forcing (ZF) Precoding Revisited



- MAC channel model: $\mathbf{y} = \mathbf{H}_A\mathbf{x}_A + \mathbf{H}_B\mathbf{x}_B + \mathbf{n}$
- Precoder at User A: $\mathbf{x}_A = \mathbf{H}_A^{-1}\mathbf{\Psi}_A\mathbf{c}_A$
- Precoder at User B: $\mathbf{x}_B = \mathbf{H}_B^{-1}\mathbf{\Psi}_B\mathbf{c}_B$
- Equivalent parallel channels for the MAC phase:

$$\mathbf{y} = \mathbf{\Psi}_A\mathbf{c}_A + \mathbf{\Psi}_B\mathbf{c}_B + \mathbf{n}$$

- When $n_T < n_R$, the right inverses $\mathbf{H}_A^{-1}$ and $\mathbf{H}_B^{-1}$ do not exist!

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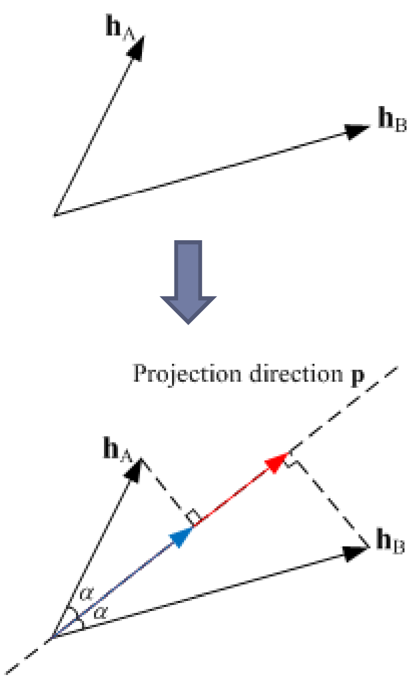
Reduced-Dimension (RD) Approach

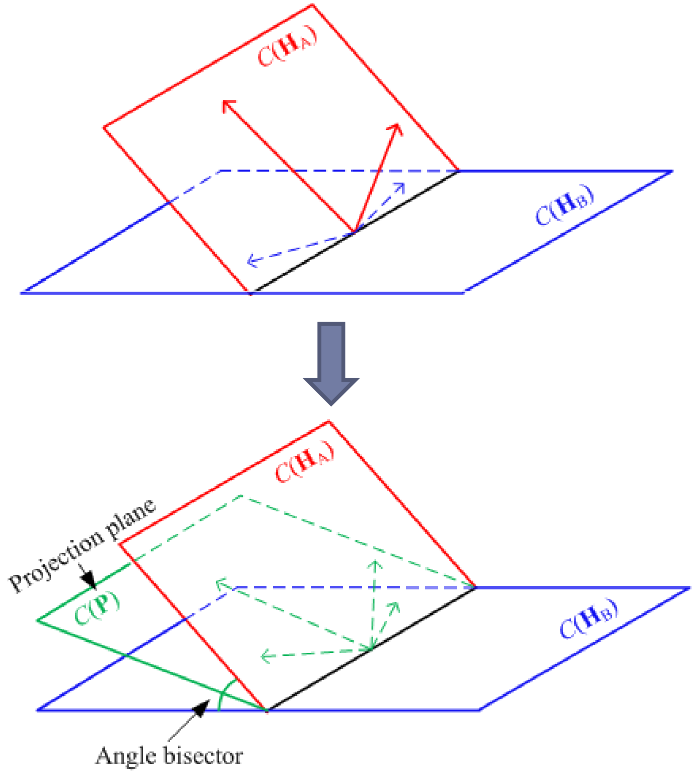
- MAC phase model: $\mathbf{y} = \mathbf{H}_A\mathbf{x}_A + \mathbf{H}_B\mathbf{x}_B + \mathbf{n}$
- Projection Matrix: $\mathbf{P}$ (which is an n_R-by-n_T matrix satisfying $\mathbf{P}^H\mathbf{P} = \mathbf{I}$)
- Effective channel: $\mathbf{P}^H\mathbf{y} = \mathbf{P}^H\mathbf{H}_A\mathbf{x}_A + \mathbf{P}^H\mathbf{H}_B\mathbf{x}_B + \mathbf{P}^H\mathbf{n}$
- Both $\mathbf{P}^H\mathbf{H}_A$ and $\mathbf{P}^H\mathbf{H}_B$ are n_T-by-n_T square matrices. Then, the previous results can be applied.
- This approach is referred to as Reduced-Dimension (RD) precoding.
- The remaining problem is how to optimize the projection matrix $\mathbf{P}$.

Geometrical Illustrations of RD Projection

- Resulting channel after projection : $\mathbf{P}^H\mathbf{y} = \mathbf{P}^H\mathbf{H}_A\mathbf{x}_A + \mathbf{P}^H\mathbf{H}_B\mathbf{x}_B + \mathbf{P}^H\mathbf{n}$
- What is the optimal projection matrix $\mathbf{P}$?



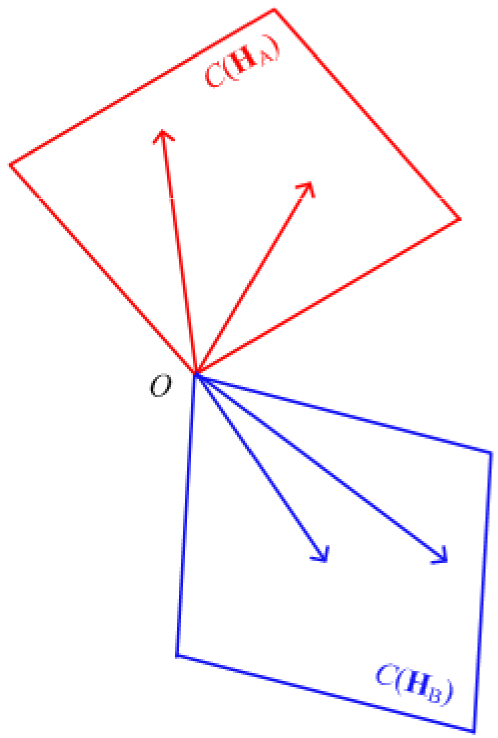
n_T = 1



n_T = 2 and n_R = 3

Geometrical Illustrations of RD Projection (Continued)

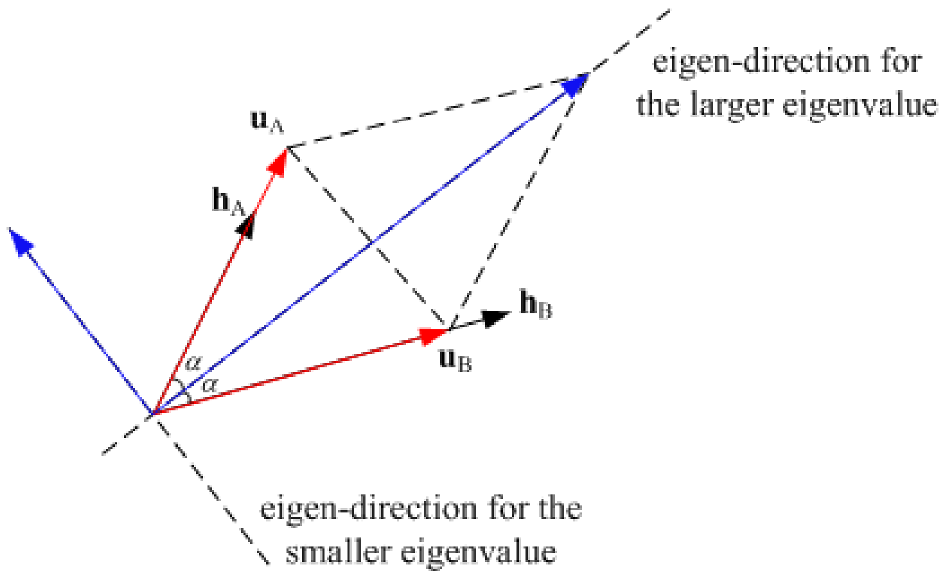
- After Projection: $\mathbf{P}^H\mathbf{y} = \mathbf{P}^H\mathbf{H}_A\mathbf{x}_A + \mathbf{P}^H\mathbf{H}_B\mathbf{x}_B + \mathbf{P}^H\mathbf{n}$
- What is the optimal $\mathbf{P}$ for the case of $n_T = 2$ and $n_R = 4$ or larger?



P?

From Geometry to Linear Algebra

- MAC channel model with $n_T = 1$: $\mathbf{y} = \mathbf{h}_A x_A + \mathbf{h}_B x_B + \mathbf{n}$
- Projection Matrix $\mathbf{P}$ reduces to a vector $\mathbf{p}$
- Resulting scalar channel: $\mathbf{p}^H\mathbf{y} = \mathbf{p}^H\mathbf{h}_A x_A + \mathbf{p}^H\mathbf{h}_B x_B + \mathbf{p}^H\mathbf{n}$



eigen-decomposition of $\mathbf{u}_A\mathbf{u}_A^H + \mathbf{u}_B\mathbf{u}_B^H$

$\mathbf{h}_A \parallel \mathbf{h}_B$: larger eigenvalue = 2

$\mathbf{h}_A \perp \mathbf{h}_B$: larger eigenvalue = 1

Optimal Projection Matrix $\mathbf{P}$

- The optimal $\mathbf{P}$ to maximize the capacity upper-bound:

$$\min_{\mathbf{P}^{\mathrm{H}}\mathbf{P}=\mathbf{I}} \sum_{m\in\{A,B\}} \log\det\left(\mathbf{I}+\frac{P_m}{n_T}\mathbf{P}^{\mathrm{H}}\mathbf{H}_m\mathbf{H}_m^{\mathrm{H}}\mathbf{P}\right)$$

- Simplification in the high-SNR regime:

$$\min_{\mathbf{P}^{\mathrm{H}}\mathbf{P}=\mathbf{I}} \sum_{m\in\{A,B\}} \log\det\left(\mathbf{P}^{\mathrm{H}}\mathbf{H}_m\mathbf{H}_m^{\mathrm{H}}\mathbf{P}\right)$$

Theorem 1: The columns of the optimal $\mathbf{P}$ are given by the eigenvectors corresponding to the n_T largest eigenvalues of

$$\mathbf{U}_{\mathrm{A}}\mathbf{U}_{\mathrm{A}}^{\mathrm{H}}+\mathbf{U}_{\mathrm{B}}\mathbf{U}_{\mathrm{B}}^{\mathrm{H}}$$

where U_{A} and U_{B} are given by the QR decomposition of

$$\mathbf{H}_{\mathrm{A}}=\mathbf{U}_{\mathrm{A}}\mathbf{R}_{\mathrm{A}} \text{ and } \mathbf{H}_{\mathrm{B}}=\mathbf{U}_{\mathrm{B}}\mathbf{R}_{\mathrm{B}}.$$

Asymptotic Analysis at High SNR

- **Theorem 2:** In the high-SNR regime, the achievable sum-rate of the proposed space-division scheme is given by

$$R^{RD} = R^{UB} - \Delta^{RD}$$

where

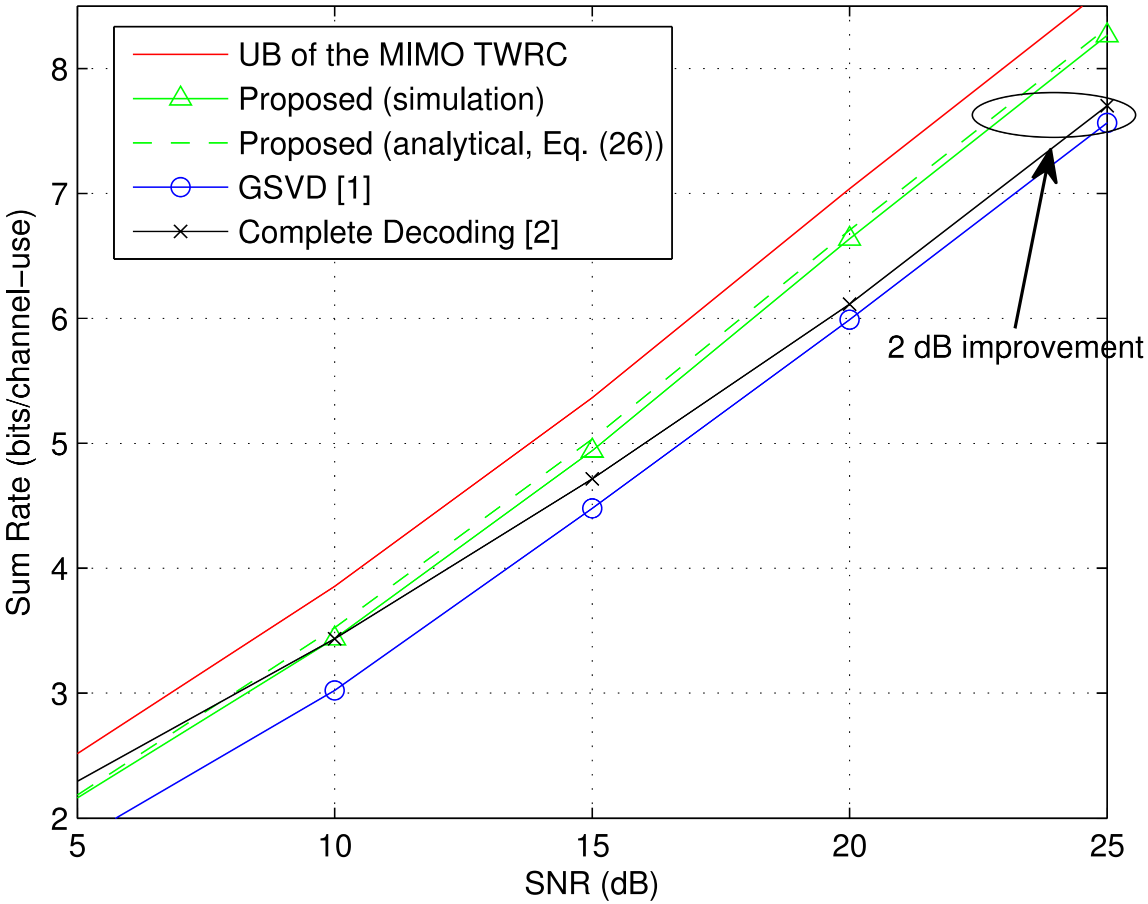
$$\Delta^{RD} = -\sum_{i=1}^{n_T} \log \frac{\lambda_i}{2}$$

is the gap to the sum-capacity upper bound, and λ_i is the ith eigenvalue of

$$\mathbf{U}_{\mathrm{A}}\mathbf{U}_{\mathrm{A}}^{\mathrm{H}} + \mathbf{U}_{\mathrm{B}}\mathbf{U}_{\mathrm{B}}^{\mathrm{H}}.$$

- **Corollary 2.1:** As each λ_i is confined in $[1, 2]$, the rate loss Δ^{RD} is at most n_{T} bits, or $1/2$ bit per user per antenna.

Average achievable sum-rates of various schemes for Rayleigh fading MIMO TWRC with $n_A = n_B = 2$ and $n_R = 4$



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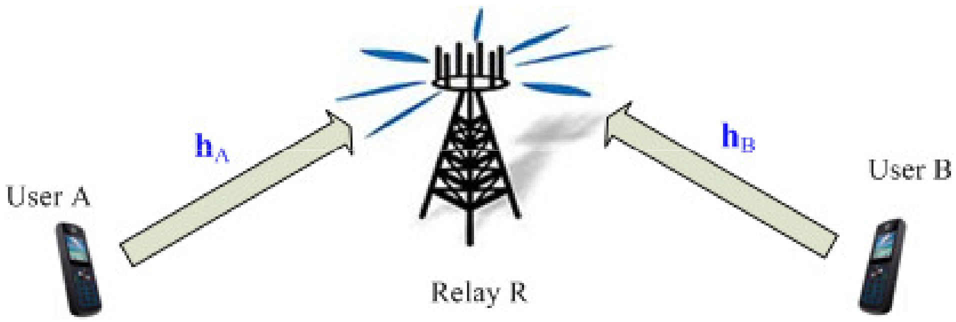
TWRC with Multi-Antenna Relay

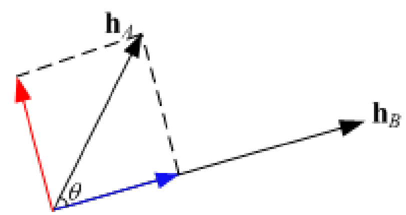
- MAC-Phase channel model: $\mathbf{y} = \mathbf{h}_A x_A + \mathbf{h}_B x_B + \mathbf{n}$



- The problem is again how to decode upon receiving **y** at the relay.

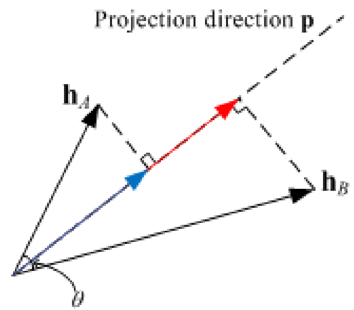
Decoding Strategies at Relay

- MAC-phase channel model: $\mathbf{y} = \mathbf{h}_A x_A + \mathbf{h}_B x_B + \mathbf{n}$
- Complete Decoding
  - Two users employ independent random coding
  - The relay decodes x_A and x_B completely as in a multiple-access channel
- PNC Decoding
  - Projection direction: $\mathbf{p}$
  - Effective SISO channel: $\mathbf{p}^H\mathbf{y} = \mathbf{p}^H\mathbf{h}_A x_A + \mathbf{p}^H\mathbf{h}_B x_B + \mathbf{p}^H\mathbf{n}$
  - Optimal projection direction for sum-rate maximization: angular bisector of $\mathbf{h}_A$ and $\mathbf{h}_B$



Project $\mathbf{h}_A$ to the orthogonal direction of $\mathbf{h}_B$

Complete decoding

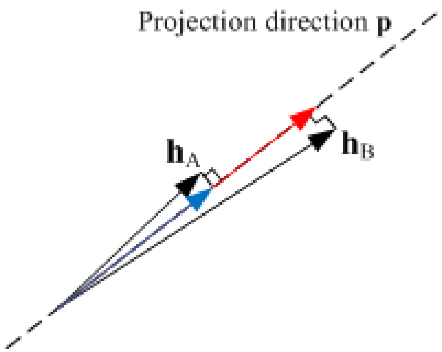


Project $\mathbf{h}_A$ and $\mathbf{h}_B$ onto a common direction $\mathbf{p}$

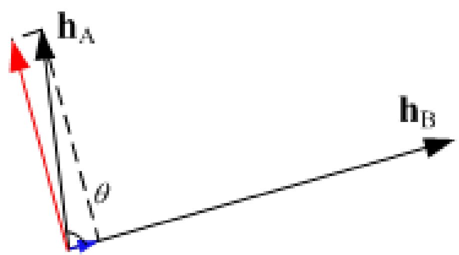
PNC decoding

PNC Decoding vs. Complete Decoding

- PNC decoding or Complete decoding
  - Depends on the angle between $\mathbf{h}_A$ and $\mathbf{h}_B$
    - $\theta \approx 0°$ → PNC decoding
    - $\theta \approx 90°$ → Complete decoding
    - Threshold of $\theta \approx 53°$ (in terms of sum-rate maximization)



PNC decoding: $\mathbf{h}_A$ // $\mathbf{h}_B$



Complete decoding: $\mathbf{h}_A \perp \mathbf{h}_B$

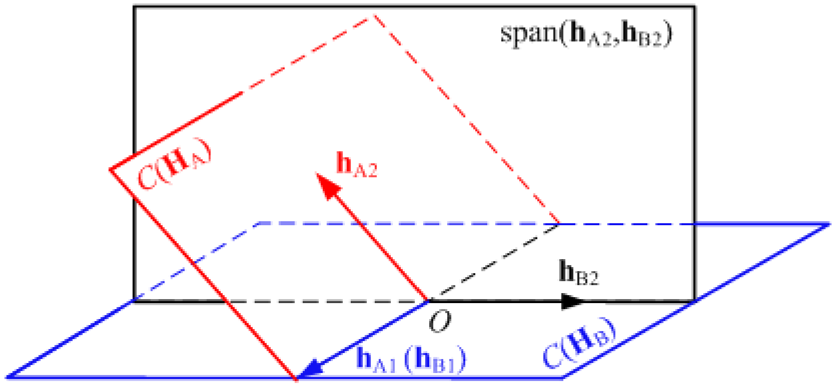
Space-Division Approach

- The main idea is to divide the joint signal space (i.e., the joint column space of $\mathbf{H}_A$ and $\mathbf{H}_B$) into two subspaces
  - S_1: contains direction pairs that are close to parallel
  - S_2: contains direction pairs that are close to orthogonal

- Space-Division Strategy
  - PNC decoding is applied to the signal subspace S_1
  - Complete decoding is applied to the signal subspace S_2

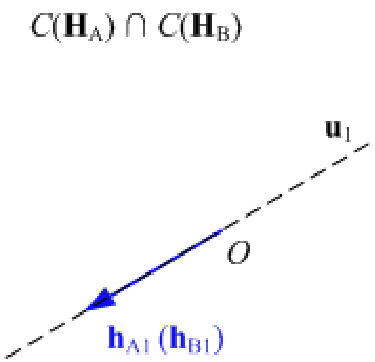
- The difficulty is how to identify those close-to-parallel or close-to-orthogonal direction pairs.

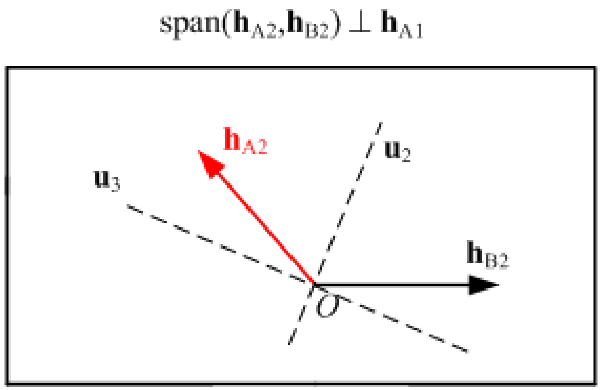
Receive-Signal Space Decomposition: $n_T = 2$ and $n_R = 3$



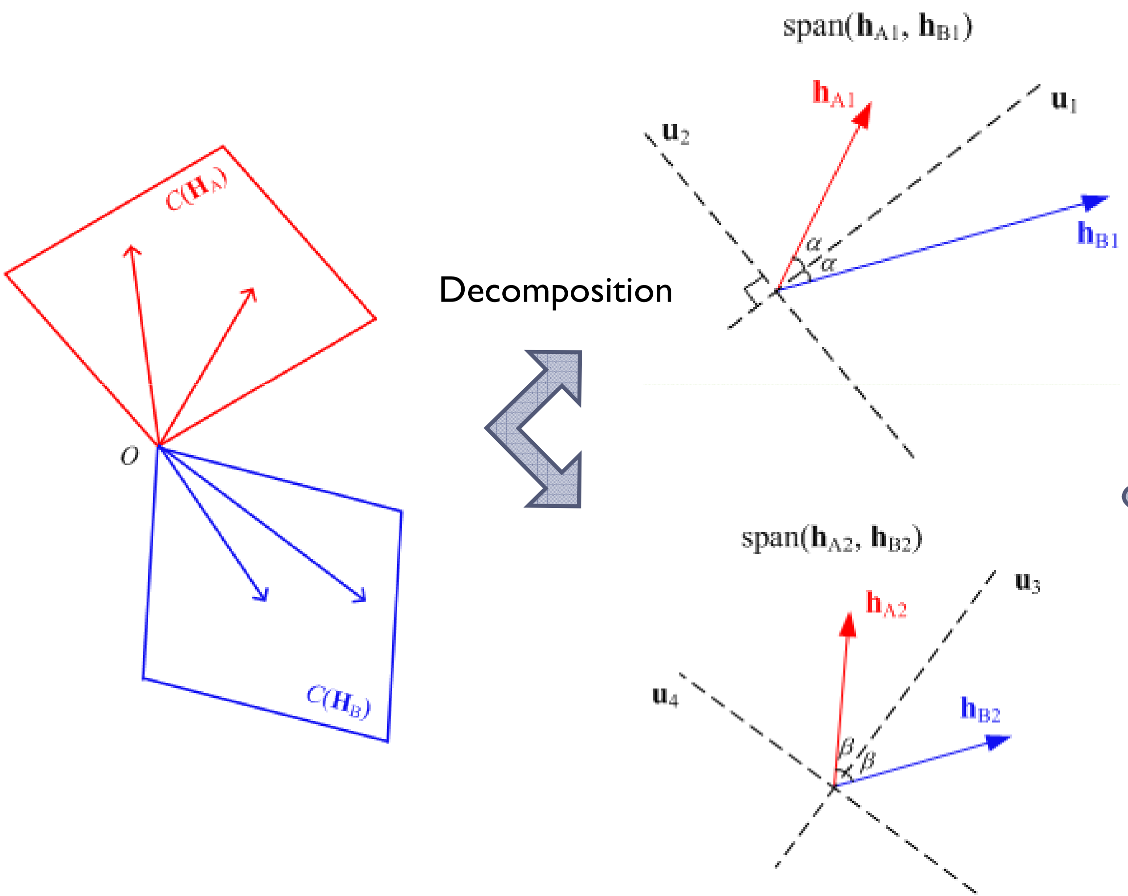
Key: $\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$ are the eigen-directions of $\mathbf{U}_A\mathbf{U}_A^H + \mathbf{U}_B\mathbf{U}_B^H$; and angles given by eigenvalues.

Decomposed into two orthogonal subspaces





Receive-Signal Space Decomposition: $n_T = 2$ and $n_R = 4$



Key: $\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$, $\mathbf{u}_4$ are the eigen-directions of $\mathbf{U}_A\mathbf{U}_A^H + \mathbf{U}_B\mathbf{U}_B^H$; and angles given by eigenvalues

Joint Channel Decomposition

Theorem 3: The channel matrices $\mathbf{H}_A$ and $\mathbf{H}_B$ can be jointly decomposed as

$$\mathbf{H}_m = \mathbf{U}\mathbf{D}_m\mathbf{G}_m,\ m \in \{A, B\}$$

where $\mathbf{G}_m$ is an n_T-by-n_T matrix,

$$\mathbf{D}_A = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{E}_A & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \quad \mathbf{D}_B = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{E}_B & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix} \quad \mathbf{E}_A = \begin{bmatrix} \sqrt{\frac{\lambda_{k+1}}{2}} & \\ \sqrt{\frac{2-\lambda_{k+1}}{2}} & \\ & \sqrt{\frac{\lambda_{k+2}}{2}} \\ & \sqrt{\frac{2-\lambda_{k+2}}{2}} \end{bmatrix} \quad \mathbf{E}_B = \begin{bmatrix} \sqrt{\frac{\lambda_{k+1}}{2}} & \\ -\sqrt{\frac{2-\lambda_{k+1}}{2}} & \\ & \sqrt{\frac{\lambda_{k+2}}{2}} \\ & -\sqrt{\frac{2-\lambda_{k+2}}{2}} \end{bmatrix}$$

the columns of $\mathbf{U}$ are given by the eigenvectors of $\mathbf{U}_A\mathbf{U}_A^H + \mathbf{U}_B\mathbf{U}_B^H$, where $\mathbf{U}_A$ and $\mathbf{U}_B$ are given by the QR decomposition of

$$\mathbf{H}_m = \mathbf{U}_m\mathbf{R}_m,\ m \in \{A, B\}.$$

Asymptotic Analysis at High SNR

- **Theorem 4:** In the high-SNR regime, the achievable sum-rate of the proposed space-division scheme is given by

$$R^{SD} = R^{UB} - \Delta^{SD}$$

where *k* is the number of PNC spatial streams, and

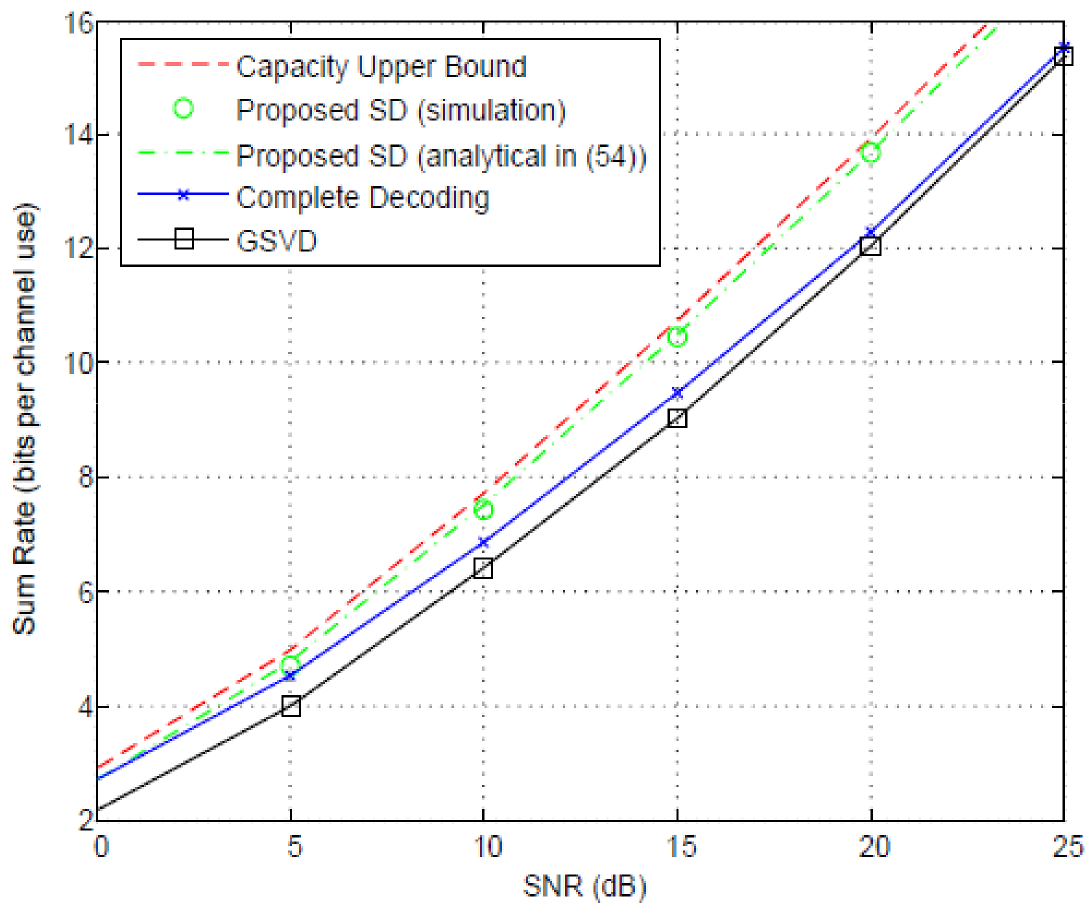
$$\Delta^{SD} = -\sum_{i=1}^{k} \log \frac{\lambda_i}{2} - \sum_{i=k+1}^{n_T} \log \sqrt{\lambda_i (2-\lambda_i)}$$

is the gap to the sum-capacity upper bound, and λ_i is the *i*th eigenvalue of

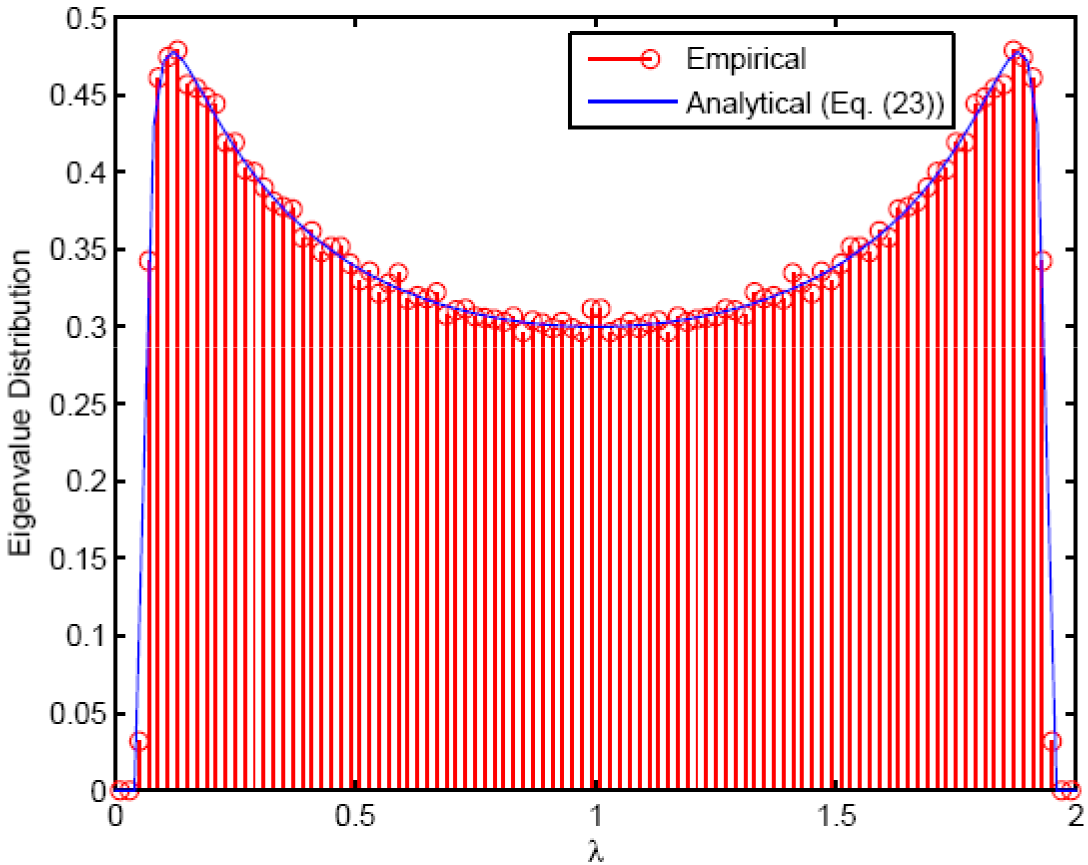
$$\mathbf{U}_A \mathbf{U}_A^H + \mathbf{U}_B \mathbf{U}_B^H.$$

- **Corollary 4.1:** The rate loss Δ^{SD} is at most $1/2\log(5/4) \approx 0.16$ bit per user per antenna.

Average achievable sum-rates of various schemes for Rayleigh fading MIMO TWRC with $n_T = 2$ and $n_R = 4$



Eigenvalue distributions of $U_A U_A^H + U_B U_B^H$, where $n_T = 400$ and $n_R = 600$. Obtained by averaging over 100 channel realizations.



Large System Analysis at High SNR

- Large System Analysis: As n_T and n_R tends to infinity, the ratio n_T/n_R tends to a constant η.

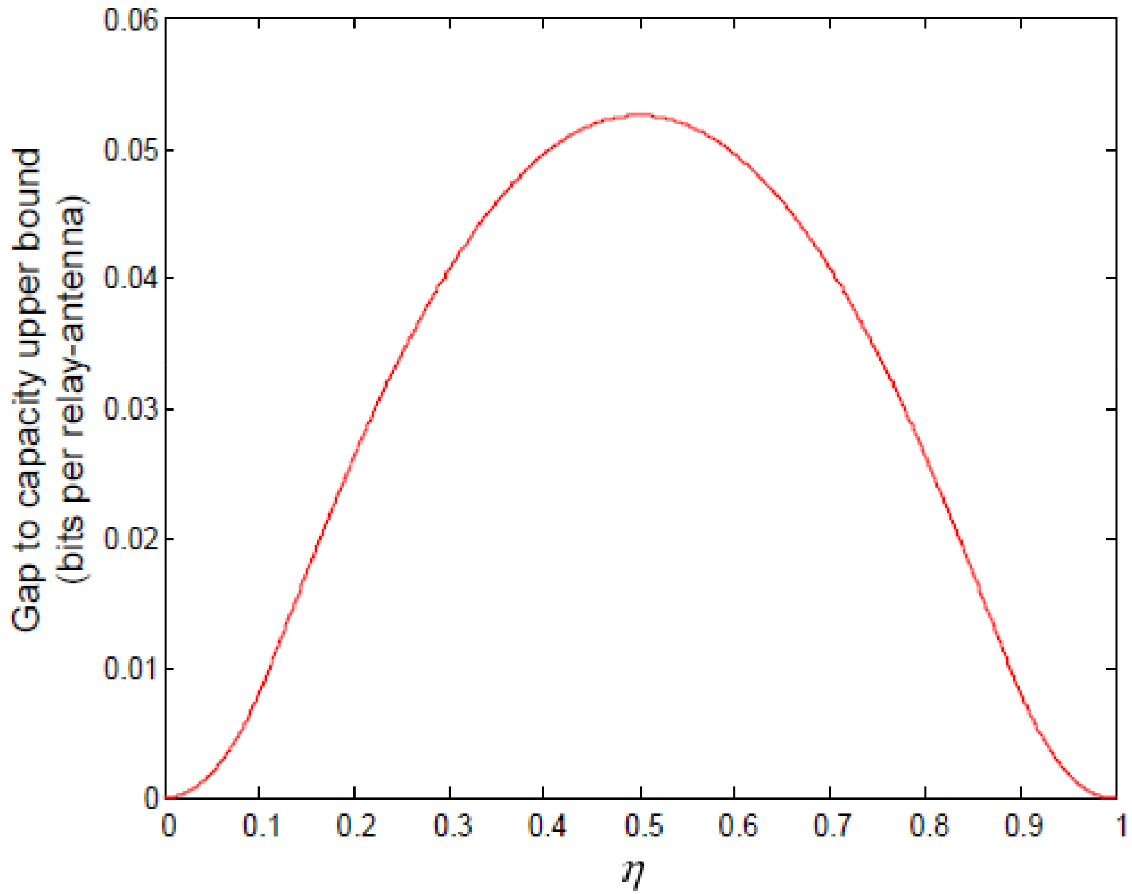
- **Theorem 5:** For Rayleigh fading, the average rate gap between the proposed SD scheme and the sum-capacity upper bound is given by

$$\lim_{n_R \to \infty} \frac{\mathrm{E}[\Delta]}{n_R} = -\left(\int_1^{8/5} \log \sqrt{\lambda(2-\lambda)} + \int_{8/5}^{2} \log \frac{\lambda}{2} \right) p(\lambda;\eta) d\lambda$$

- The above normalized rate gap is maximized at $\eta = 1/2$, with the maximum given by

$$\lim_{n_R \to \infty} \frac{\mathrm{E}[\Delta]}{n_R} = -\frac{1}{\pi}\left(\int_1^{8/5} \frac{\log \sqrt{\lambda(2-\lambda)}}{\sqrt{2\lambda-\lambda^2}} \mathrm{d}\lambda + \int_{8/5}^{2} \frac{\log(\lambda/2)}{\sqrt{2\lambda-\lambda^2}} \mathrm{d}\lambda \right) = 0.053 \text{ bit}$$

Average Rate Gap against $\eta = n_T/n_R$



Outline

- Background and Motivation
- MIMO TWRC Model and Previously
- Reduced-Dimension Precoding
- Space-Division Approach
- Conclusions and Future Work

Conclusions

- We propose a PNC technique based on reduced-dimension (RD) precoding technique for MIMO TWRC.
- To reduce the gap to the capacity, we further propose a space-division (SD) approach for MIMO TWRC.
- At high SNR, the proposed SD scheme approach the sum-rate capacity of MIMO TWRC within $1/2\log(5/4) \approx 0.161$ bit per user-antenna.
- In Rayleigh fading channels, the average rate gap is bounded by 0.053 bit per relay-antenna, which occurs at $n_T/n_R = 0.5$.

Related Publications

- Tao Yang, Xiaojun Yuan, and Iain B. Collings, "Reduced-dimension cooperative precoding for MIMO two-way relay channels", *IEEE Trans. Wireless Commun., vol. 11, no. 11, Nov 2012.*
- Xiaojun Yuan, Tao Yang, and Iain B. Collings, "MIMO two-way relaying: A space-division approach", *IEEE Trans. Information Theory, submitted, 2012, second revision.*

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Future Work

- PNC design for multi-user MIMO TWRC
- Analogue network coding (ANC) for multi-user MIMO TWRC
- Practical channel coding design for MIMO TWRC
- Impact of channel uncertainty on PNC for MIMO TWRC

Thank you !