List Decoding, Combinatorial Aspects

Vladimir Blinovsky[1]

[1] Institute of Information Transmission Problems

August, 2012

Outline

- History: List Decoding

Outline

- History: List Decoding
- Coding Bounds, Binary Case

Outline

- History: List Decoding
- Coding Bounds, Binary Case
- Coding Bounds, $q-$ary case

Outline

- History: List Decoding
- Coding Bounds, Binary Case
- Coding Bounds, $q-$ary case
- Bounds for Reliability Function

Outline

- History: List Decoding
- Coding Bounds, Binary Case
- Coding Bounds, $q-$ary case
- Bounds for Reliability Function
- Bounds in Euclidean Space

Outline

- History: List Decoding
- Coding Bounds, Binary Case
- Coding Bounds, $q-$ary case
- Bounds for Reliability Function
- Bounds in Euclidean Space
- Open Problems

Outline

- History: List Decoding
- Coding Bounds, Binary Case
- Coding Bounds, $q-$ary case
- Bounds for Reliability Function
- Bounds in Euclidean Space
- Open Problems
- Conclusion

History: List Decoding

Elias, 1957
Wozencraft, 1958

Coding Bounds, Binary Case

$$\tau = \sum_{i=1}^{[L/2]} \frac{\binom{2i-2}{i-1}}{i} (\lambda(1-\lambda))^i,$$
$$R = 1 - H(\lambda),$$
$$H(x) = -x \log x - (1-x) \log(1-x).$$

Coding Bounds, $q-$ary Case

$$\tau = \sum_{j_i \ge 0,\ \sum_{i=1}^{q} j_i = L+1} \binom{L+1}{j_1, \dots, j_q} \times$$

$$\left(1 - \frac{\max\{j_1, \dots, j_q\}}{L+1}\right) \left(\frac{\lambda}{q-1}\right)^{L+1-j_q} (1-\lambda)^{j_q},$$

$$R = 1 - H_q(\lambda),$$

$$H_q(x) = -x \log_q x - (1-x)\log_q(1-x) + x\log_q(q-1).$$

List Plotkin Bound

$$\frac{\tau}{\tau_0(L)} \leq \frac{M^L}{(M-1)\dots(M-L)}.$$

Bounds for Reliability Function

$$E_L(0) = - \min_{\{p_{i_j}: \sum_{t=1}^{|X|} P_t = 1} \sum_{(i_1, \ldots, i_{L+1})} p_{i_1} \ldots p_{i_{L+1}} \times$$

$$\ln \left(\sum_{j=1}^{|Y|} (p(j|i_1) \ldots p(j|i_{L+1}))^{\frac{1}{L+1}} \right).$$

Using the proof technique of this relation can be essentially improved the Shannon- Gallager- Berlecamp bound for the deviation reliability function for usual decoding for finite code volume M from its optimal limit.

In Binary symmetric Channel

$$E_L(0) = -\frac{1}{2^{L+1}} \sum_{i=0}^{L+1} \binom{L+1}{i} \times$$
$$\ln(p^{i/(L+1)}(1-p)^{1-i/(L+1)} + p^{1-i/(L+1)}(1-p)^{i/(L+1)}).$$

Bounds in Euclidean Space

$$r \leq \frac{1}{2} \ln \frac{L}{(L+1)t_L^2},$$

$$R \geq \frac{1}{2} \ln \frac{L}{(L+1)t_L^2} +$$

$$+\frac{1}{2L} \ln \frac{1}{(L+1)(1-t_L^2)}.$$

At low rates these bounds can be improved such that they touch the vertical axis (this is not proved in discrete case).

Open Problems

- Improve upper bound in Euclidean Space
- Linear Programming Bound for List Decoding Codes
- List Decoding for Quantum Channels
- Optimal Multiple packing of 3-dimensional Euclidean space

Thank you for your attention!