6.2 Strong Typicality vs Weak Typicality
Summary

- Weak typicality: empirical entropy $\approx H(X)$

- Strong typicality

- Both have AEP, but strong typicality has stronger conditional asymptotic properties (Theorem 6.10).

- Strong typicality works only for finite alphabet, i.e., $|X| < 1$, but weak typicality works for any countable alphabet.
Summary

- Weak typicality: empirical entropy $\approx H(X)$
- Strong typicality: empirical distribution $\sim p(x)$
Summary

- Weak typicality: empirical entropy $\approx H(X)$
- Strong typicality: empirical distribution $\sim p(x)$
- Strong typicality $\Rightarrow$ weak typicality (Proposition 6.5)

Both have AEP, but strong typicality has stronger conditional asymptotic properties (Theorem 6.10).

Strong typicality works only for finite alphabet, i.e., $|X| < 1$, but weak typicality works for any countable alphabet.
Summary

- Weak typicality: empirical entropy $\approx H(X)$
- Strong typicality: empirical distribution $\sim p(x)$
- Strong typicality $\Rightarrow$ weak typicality (Proposition 6.5)
- Weak typicality $\not\Rightarrow$ strong typicality (to be discussed)
Summary

• Weak typicality: empirical entropy $\approx H(X)$
• Strong typicality: empirical distribution $\sim p(x)$
• Strong typicality $\Rightarrow$ weak typicality (Proposition 6.5)
• Weak typicality $\not\Rightarrow$ strong typicality (to be discussed)
• Both have AEP, but strong typicality has stronger conditional asymptotic properties (Theorem 6.10).
Summary

- Weak typicality: empirical entropy $\approx H(X)$
- Strong typicality: empirical distribution $\sim p(x)$
- Strong typicality $\Rightarrow$ weak typicality (Proposition 6.5)
- Weak typicality $\not\Rightarrow$ strong typicality (to be discussed)
- Both have AEP, but strong typicality has stronger conditional asymptotic properties (Theorem 6.10).
- Strong typicality works only for finite alphabet, i.e., $|\mathcal{X}| < \infty$, but weak typicality works for any countable alphabet.
Strong Typicality $\Rightarrow$ Weak Typicality

Proposition 6.5  For any $x \in X^n$, if $x \in T^n_{[X] \delta}$, then $x \in W^n_{[X] \eta}$, where $\eta \to 0$ as $\delta \to 0$.

Proof Idea

- By strong AEP and the definition of weak typicality.
Proposition 6.5 For any $x \in \mathcal{X}^n$, if $x \in T^n_{[X] \delta}$, then
\[ x \in W^n_{[X] \eta}, \] where $\eta \rightarrow 0$ as $\delta \rightarrow 0$.

Proof
Proposition 6.5 For any $x \in X^n$, if $x \in T^n_{[X] \delta}$, then $x \in W^n_{[X] \eta}$, where $\eta \to 0$ as $\delta \to 0$.

Proof
1. If $x \in T^n_{[X] \delta}$, by Property 1 of strong AEP, we have
Proposition 6.5  For any $x \in X^n$, if $x \in T^n_{[X] \delta}$, then $x \in W^n_{[X] \eta}$, where $\eta \to 0$ as $\delta \to 0$.

Proof
1. If $x \in T^n_{[X] \delta}$, by Property 1 of strong AEP, we have

Theorem 6.2 (Strong AEP) There exists $\eta > 0$ such that $\eta \to 0$ as $\delta \to 0$, and the following hold:

1) If $x \in T^n_{[X] \delta}$, then

$$2^{-n(H(X)+\eta)} \leq p(x) \leq 2^{-n(H(X)-\eta)}.$$
Proposition 6.5 For any $x \in \mathcal{X}^n$, if $x \in T^n[X]^{\delta}$, then $x \in W^n[X]^{\eta}$, where $\eta \to 0$ as $\delta \to 0$.

Proof
1. If $x \in T^n[X]^{\delta}$, by Property 1 of strong AEP, we have

$$2^{-n(H(X)+\eta)} \leq p(x) \leq 2^{-n(H(X)-\eta)}.$$

Theorem 6.2 (Strong AEP) There exists $\eta > 0$ such that $\eta \to 0$ as $\delta \to 0$, and the following hold:

1) If $x \in T^n[X]^{\delta}$, then

$$2^{-n(H(X)+\eta)} \leq p(x) \leq 2^{-n(H(X)-\eta)}.$$
Proposition 6.5  For any $x \in X^n$, if $x \in T^n_{[X]\delta}$, then $x \in W^n_{[X]\eta}$, where $\eta \to 0$ as $\delta \to 0$.

Proof
1. If $x \in T^n_{[X]\delta}$, by Property 1 of strong AEP, we have

$$2^{-n(H(X)+\eta)} \leq p(x) \leq 2^{-n(H(X)-\eta)}.$$  

2. This is equivalent to

$$H(X) - \eta \leq -\frac{1}{n} \log p(x) \leq H(X) + \eta,$$

Theorem 6.2 (Strong AEP)  There exists $\eta > 0$ such that $\eta \to 0$ as $\delta \to 0$, and the following hold:

1) If $x \in T^n_{[X]\delta}$, then

$$2^{-n(H(X)+\eta)} \leq p(x) \leq 2^{-n(H(X)-\eta)}.$$
Proposition 6.5 For any $x \in X^n$, if $x \in T^n_{[X] \delta}$, then $x \in W^n_{[X] \eta}$, where $\eta \to 0$ as $\delta \to 0$.

Proof
1. If $x \in T^n_{[X] \delta}$, by Property 1 of strong AEP, we have

$$2^{-n(H(X)+\eta)} \leq p(x) \leq 2^{-n(H(X)-\eta)}.$$  

2. This is equivalent to

$$H(X) - \eta \leq - \frac{1}{n} \log p(x) \leq H(X) + \eta,$$

where $\eta \to 0$ as $\delta \to 0$ as asserted by the strong AEP.

Theorem 6.2 (Strong AEP) There exists $\eta > 0$ such that $\eta \to 0$ as $\delta \to 0$, and the following hold:

1) If $x \in T^n_{[X] \delta}$, then

$$2^{-n(H(X)+\eta)} \leq p(x) \leq 2^{-n(H(X)-\eta)}.$$
Proposition 6.5  For any $x \in \mathcal{X}^n$, if $x \in T_n^{[X]\delta}$, then $x \in W_n^{[X]\eta}$, where $\eta \to 0$ as $\delta \to 0$.

Proof
1. If $x \in T_n^{[X]\delta}$, by Property 1 of strong AEP, we have

$$2^{-n(H(X)+\eta)} \leq p(x) \leq 2^{-n(H(X)-\eta)}.$$ 

2. This is equivalent to

$$H(X) - \eta \leq -\frac{1}{n} \log p(x) \leq H(X) + \eta,$$

where $\eta \to 0$ as $\delta \to 0$ as asserted by the strong AEP.

3. Then $x \in W_n^{[X]\eta}$ by Definition 5.2. The proposition is proved.

Theorem 6.2 (Strong AEP) There exists $\eta > 0$ such that $\eta \to 0$ as $\delta \to 0$, and the following hold:

1) If $x \in T_n^{[X]\delta}$, then

$$2^{-n(H(X)+\eta)} \leq p(x) \leq 2^{-n(H(X)-\eta)}.$$
Proposition 6.5  For any $x \in X^n$, if $x \in T^n_{[X]\delta}$, then $x \in W^n_{[X]\eta}$, where $\eta \to 0$ as $\delta \to 0$.

Proof
1. If $x \in T^n_{[X]\delta}$, by Property 1 of strong AEP, we have 

$$2^{-n(H(X)+\eta)} \leq p(x) \leq 2^{-n(H(X)-\eta)}.$$

2. This is equivalent to

$$H(X) - \eta \leq -\frac{1}{n} \log p(x) \leq H(X) + \eta,$$

where $\eta \to 0$ as $\delta \to 0$ as asserted by the strong AEP.

3. Then $x \in W^n_{[X]\eta}$ by Definition 5.2. The proposition is proved.

Theorem 6.2 (Strong AEP)  There exists $\eta > 0$ such that $\eta \to 0$ as $\delta \to 0$, and the following hold:

1) If $x \in T^n_{[X]\delta}$, then

$$2^{-n(H(X)+\eta)} \leq p(x) \leq 2^{-n(H(X)-\eta)}.$$

Definition 5.2  The weakly typical set $W^n_{[X]\varepsilon}$ with respect to $p(x)$ is the set of sequences $x = (x_1, x_2, \ldots, x_n) \in X^n$ such that 

$$H(X) - \varepsilon \leq -\frac{1}{n} \log p(x) \leq H(X) + \varepsilon,$$
Weak Typicality $\not\Rightarrow$ Strong Typicality
Weak Typicality $\nRightarrow$ Strong Typicality

1. Consider $X$ with distribution $p$ such that

$$p(0) = 0.5, p(1) = 0.25, p(2) = 0.25.$$
Weak Typicality $\nRightarrow$ Strong Typicality

1. Consider $X$ with distribution $p$ such that

$$p(0) = 0.5, \ p(1) = 0.25, \ p(2) = 0.25.$$
Weak Typicality $\nRightarrow$ Strong Typicality

1. Consider $X$ with distribution $p$ such that

\[ p(0) = 0.5, \quad p(1) = 0.25, \quad p(2) = 0.25. \]

2. Consider a sequence $x$ of length $n$ and let

\[ q(x) = n^{-1}N(x; x) \]

be the relative frequency of occurrence of symbol $x$ in $x$, $x = 0, 1, 2$. 

This can be satisfied by choosing $q(i) = p(i)$ for all $i$. 

Alternatively, we can choose $q(0) = 0.5$, $q(1) = 0.5$, $q(2) = 0.5$. 

With such a choice of $\{q(i)\}$, the sequence $x$ is weakly typical with respect to $p$ because (1) and (2) are evaluated to the same value which implies empirical entropy $\approx H(X)$, but obviously not strongly typical with respect to $p$, because $p \not\approx q$. 

Weak Typicality $\nRightarrow$ Strong Typicality
1. Consider $X$ with distribution $p$ such that

$$p(0) = 0.5, \quad p(1) = 0.25, \quad p(2) = 0.25.$$ 

2. Consider a sequence $x$ of length $n$ and let

$$q(x) = n^{-1} N(x; x)$$

be the relative frequency of occurrence of symbol $x$ in $x$, $x = 0, 1, 2$.

3. In order for the sequence $x$ to be weakly typical, we need the empirical entropy to be close to $H(X)$:

$$\frac{1}{n} \log p(x) = \frac{1}{n} \left[ N(0; x) \log p(0) + N(1; x) \log p(1) + N(2; x) \log p(2) \right]$$

This can be satisfied by choosing $q(i) = p(i)$ for all $i$.

4. Alternatively, we can choose $q(0) = 0.5, q(1) = 0.25, q(2) = 0.25$.

5. With such a choice of $\{q(i)\}$, the sequence $x$ is weakly typical with respect to $p$ because (1) and (2) are evaluated to the same value which implies empirical entropy $\approx H(X)$, but obviously not strongly typical with respect to $p$, because $p \not\approx q$.

Weak Typicality $\not\Rightarrow$ Strong Typicality
Weak Typicality $\not\Rightarrow$ Strong Typicality

1. Consider $X$ with distribution $p$ such that
\[ p(0) = 0.5, \quad p(1) = 0.25, \quad p(2) = 0.25. \]

2. Consider a sequence $x$ of length $n$ and let
\[ q(x) = n^{-1} N(x; x) \]
be the relative frequency of occurrence of symbol $x$ in $x$, $x = 0, 1, 2$.

3. In order for the sequence $x$ to be weakly typical, we need the empirical entropy to be close to $H(X)$:
\[
\frac{1}{n} \log p(x) \quad \text{(empirical entropy)}
\]
Weak Typicality $\nRightarrow$ Strong Typicality

1. Consider $X$ with distribution $p$ such that

\[ p(0) = 0.5, \quad p(1) = 0.25, \quad p(2) = 0.25. \]

2. Consider a sequence $x$ of length $n$ and let $q(x) = n^{-1} N(x; x)$ be the relative frequency of occurrence of symbol $x$ in $x$, $x = 0, 1, 2$.

3. In order for the sequence $x$ to be weakly typical, we need the empirical entropy to be close to $H(X)$:

\[
\frac{1}{n} \log p(x) \quad \text{(empirical entropy)}
\]

\[
= - \frac{1}{n} \log \prod_{k=1}^{n} p(x_k)
\]
Weak Typicality $\not\Rightarrow$ Strong Typicality

1. Consider $X$ with distribution $p$ such that

$$p(0) = 0.5, \ p(1) = 0.25, \ p(2) = 0.25.$$  

2. Consider a sequence $x$ of length $n$ and let

$$q(x) = n^{-1} N(x; x)$$
be the relative frequency of occurrence of symbol $x$ in $x$, $x = 0, 1, 2$.

3. In order for the sequence $x$ to be weakly typical, we need the empirical entropy to be close to $H(X)$:

$$-\frac{1}{n} \log p(x) \quad \text{(empirical entropy)}$$

$$= -\frac{1}{n} \log \prod_{k=1}^{n} p(x_k)$$

$$= -\frac{1}{n} \sum_{k=1}^{n} \log p(x_k)$$

4. Alternatively, we can choose $q(0) = 0.5, q(1) = 0.25, q(2) = 0.25$.

5. With such a choice of $\{q(i)\}$, the sequence $x$ is weakly typical with respect to $p$ because (1) and (2) are evaluated to the same value which implies $\text{empirical entropy} \approx H(X)$, but obviously not strongly typical with respect to $p$, because $p \not\approx q$. Weak Typicality $\not\Rightarrow$ Strong Typicality
Weak Typicality $\not\Rightarrow$ Strong Typicality

1. Consider $X$ with distribution $p$ such that

\[ p(0) = 0.5, \quad p(1) = 0.25, \quad p(2) = 0.25. \]

2. Consider a sequence $x$ of length $n$ and let $q(x) = n^{-1} N(x; x)$ be the relative frequency of occurrence of symbol $x$ in $x$, $x = 0, 1, 2$.

3. In order for the sequence $x$ to be weakly typical, we need the empirical entropy to be close to $H(X)$:

\[
- \frac{1}{n} \log p(x) \quad \text{(empirical entropy)}
\]

\[
= - \frac{1}{n} \log \prod_{k=1}^{n} p(x_k)
\]

\[
= - \frac{1}{n} \sum_{k=1}^{n} \log p(x_k)
\]

\[
= - \frac{1}{n} \left[ N(0; x) \log p(0) + N(1; x) \log p(1) + N(2; x) \log p(2) \right]
\]

This can be satisfied by choosing $q(i) = p(i)$ for all $i$.

4. Alternatively, we can choose $q(0) = 0.5, q(1) = 0.5, q(2) = 0.25$.

5. With such a choice of $\{q(i)\}$, the sequence $x$ is weakly typical with respect to $p$ because (1) and (2) are evaluated to the same value which implies $H(X)$, but obviously not strongly typical with respect to $p$.

Weak Typicality $\not\Rightarrow$ Strong Typicality
Weak Typicality $\not\Rightarrow$ Strong Typicality

1. Consider $X$ with distribution $p$ such that

$$p(0) = 0.5, \; p(1) = 0.25, \; p(2) = 0.25.$$ 

2. Consider a sequence $x$ of length $n$ and let $q(x) = n^{-1} N(x; x)$ be the relative frequency of occurrence of symbol $x$ in $x$, $x = 0, 1, 2$.

3. In order for the sequence $x$ to be weakly typical, we need the empirical entropy to be close to $H(X)$:

$$\frac{1}{n} \log p(x) \quad \text{(empirical entropy)}$$ 

$$= - \frac{1}{n} \log \prod_{k=1}^{n} p(x_k)$$ 

$$= - \frac{1}{n} \sum_{k=1}^{n} \log p(x_k)$$ 

$$= - \frac{1}{n} [N(0; x) \log p(0) + N(1; x) \log p(1) + N(2; x) \log p(2)]$$ 

$$= - \frac{N(0; x)}{n} \log p(0) - \frac{N(1; x)}{n} \log p(1) - \frac{N(2; x)}{n} \log p(2)$$
Weak Typicality $\nrightarrow$ Strong Typicality

1. Consider $X$ with distribution $p$ such that

\[ p(0) = 0.5, \ p(1) = 0.25, \ p(2) = 0.25. \]

2. Consider a sequence $x$ of length $n$ and let

\[ q(x) = n^{-1} N(x; x) \]

be the relative frequency of occurrence of symbol $x$ in $x$, $x = 0, 1, 2$.

3. In order for the sequence $x$ to be weakly typical, we need the empirical entropy to be close to $H(X)$:

\[
- \frac{1}{n} \log p(x) \quad \text{(empirical entropy)}
\]

\[
= - \frac{1}{n} \log \prod_{k=1}^{n} p(x_k)
\]

\[
= - \frac{1}{n} \sum_{k=1}^{n} \log p(x_k)
\]

\[
= - \frac{1}{n} \left[ N(0; x) \log p(0) + N(1; x) \log p(1) + N(2; x) \log p(2) \right]
\]

\[
= - \frac{N(0; x)}{n} \log p(0) - \frac{N(1; x)}{n} \log p(1) - \frac{N(2; x)}{n} \log p(2)
\]
1. Consider $X$ with distribution $p$ such that

$$p(0) = 0.5, \quad p(1) = 0.25, \quad p(2) = 0.25.$$ 

2. Consider a sequence $x$ of length $n$ and let

$$q(x) = n^{-1} N(x; x)$$

be the relative frequency of occurrence of symbol $x$ in $x$, $x = 0, 1, 2$.

3. In order for the sequence $x$ to be weakly typical, we need the empirical entropy to be close to $H(X)$:

$$- \frac{1}{n} \log p(x) \quad \text{(empirical entropy)}$$

$$= - \frac{1}{n} \log \prod_{k=1}^{n} p(x_k)$$

$$= - \frac{1}{n} \sum_{k=1}^{n} \log p(x_k)$$

$$= - \frac{1}{n} [N(0; x) \log p(0) + N(1; x) \log p(1) + N(2; x) \log p(2)]$$

$$= - \frac{N(0; x)}{n} \log p(0) - \frac{N(1; x)}{n} \log p(1) - \frac{N(2; x)}{n} \log p(2)$$

4. Alternatively, we can choose $q(0) = 0.5, q(1) = 0.25, q(2) = 0.25$.

With such a choice of $\{q(i)\}$, the sequence $x$ is weakly typical with respect to $p$ because (1) and (2) are evaluated to the same value which implies empirical entropy $\Rightarrow H(X)$, but obviously not strongly typical with respect to $p$, because $p \not\Rightarrow q$.

Weak Typicality $\not\Rightarrow$ Strong Typicality
1. Consider $X$ with distribution $p$ such that
   \[ p(0) = 0.5, \quad p(1) = 0.25, \quad p(2) = 0.25. \]

2. Consider a sequence $x$ of length $n$ and let
   \[ q(x) = n^{-1} N(x; x) \]
   be the relative frequency of occurrence of symbol $x$ in $x$, $x = 0, 1, 2$.

3. In order for the sequence $x$ to be weakly typical, we need the empirical entropy to be close to $H(X)$:

   \[
   \frac{1}{n} \log p(x) \text{ (empirical entropy)}
   \]
   \[
   = \frac{1}{n} \log \prod_{k=1}^{n} p(x_k)
   \]
   \[
   = \frac{1}{n} \sum_{k=1}^{n} \log p(x_k)
   \]
   \[
   = - \frac{1}{n} \left[ N(0; x) \log p(0) + N(1; x) \log p(1) + N(2; x) \log p(2) \right]
   \]
   \[
   = - \frac{N(0; x)}{n} \log p(0) - \frac{N(1; x)}{n} \log p(1) - \frac{N(2; x)}{n} \log p(2)
   \]
   \[
   = -q(0) \log 0.5 - q(1) \log 0.25 - q(2) \log 0.25 \quad (1)
   \]

4. Alternatively, we can choose $q(0) = 0.5, q(1) = 0.25, q(2) = 0.25$.

With such a choice of $\{q(i)\}$, the sequence $x$ is weakly typical with respect to $p$ because (1) and (2) are evaluated to the same value which implies empirical entropy $\approx H(X)$, but obviously not strongly typical with respect to $p$, because $p \not\approx q$.

Weak Typicality $\not\Rightarrow$ Strong Typicality
Weak Typicality $\not\Rightarrow$ Strong Typicality

1. Consider $X$ with distribution $p$ such that

$$p(0) = 0.5, \quad p(1) = 0.25, \quad p(2) = 0.25.$$ 

2. Consider a sequence $\mathbf{x}$ of length $n$ and let

$$q(x) = n^{-1} N(x; \mathbf{x})$$
be the relative frequency of occurrence of symbol $x$ in $\mathbf{x}$, $x = 0, 1, 2$. 

3. In order for the sequence $\mathbf{x}$ to be weakly typical, we need the empirical entropy to be close to $H(X)$:

$$-\frac{1}{n} \log p(\mathbf{x}) \quad (\text{empirical entropy})$$

$$= -\frac{1}{n} \log \prod_{k=1}^{n} p(x_k)$$

$$= -\frac{1}{n} \sum_{k=1}^{n} \log p(x_k)$$

$$= -\frac{1}{n} \left[ N(0; \mathbf{x}) \log p(0) + N(1; \mathbf{x}) \log p(1) + N(2; \mathbf{x}) \log p(2) \right]$$

$$= -\frac{N(0; \mathbf{x})}{n} \log p(0) - \frac{N(1; \mathbf{x})}{n} \log p(1) - \frac{N(2; \mathbf{x})}{n} \log p(2)$$

$$= -q(0) \log 0.5 - q(1) \log 0.25 - q(2) \log 0.25 \quad (1)$$

$$\approx H(X)$$
Weak Typicality $\nRightarrow$ Strong Typicality

1. Consider $X$ with distribution $p$ such that

$$p(0) = 0.5, \ p(1) = 0.25, \ p(2) = 0.25.$$ 

2. Consider a sequence $x$ of length $n$ and let

$$q(x) = n^{-1} N(x; x)$$

be the relative frequency of occurrence of symbol $x$ in $x$, $x = 0, 1, 2$.

3. In order for the sequence $x$ to be weakly typical, we need the empirical entropy to be close to $H(X)$:

$$-\frac{1}{n} \log p(x) \quad \text{(empirical entropy)}$$

$$= -\frac{1}{n} \log \prod_{k=1}^{n} p(x_k)$$

$$= -\frac{1}{n} \sum_{k=1}^{n} \log p(x_k)$$

$$= -\frac{1}{n} [N(0; x) \log p(0) + N(1; x) \log p(1) + N(2; x) \log p(2)]$$

$$= -\frac{N(0; x)}{n} \log p(0) - \frac{N(1; x)}{n} \log p(1) - \frac{N(2; x)}{n} \log p(2)$$

$$= -q(0) \log 0.5 - q(1) \log 0.25 - q(2) \log 0.25$$

$$\approx H(X)$$

$$= -p(0) \log 0.5 - p(1) \log 0.25 - p(2) \log 0.25$$

4. Alternatively, we can choose $q(0) = 0.5$, $q(1) = 0.25$, $q(2) = 0.25$.

5. With such a choice of $\{q(i)\}$, the sequence $x$ is weakly typical with respect to $p$ because (1) and (2) are evaluated to the same value which implies $\text{empirical entropy} \approx H(X)$, but obviously not strongly typical with respect to $p$, because $p \neq q$. Weak Typicality $\nRightarrow$ Strong Typicality.
Weak Typicality \not\Rightarrow Strong Typicality

1. Consider $X$ with distribution $p$ such that

$$p(0) = 0.5, \ p(1) = 0.25, \ p(2) = 0.25.$$  

2. Consider a sequence $\mathbf{x}$ of length $n$ and let $q(x) = n^{-1} N(x; \mathbf{x})$ be the relative frequency of occurrence of symbol $x$ in $\mathbf{x}$, $x = 0, 1, 2$.

3. In order for the sequence $\mathbf{x}$ to be weakly typical, we need the empirical entropy to be close to $H(X)$:

$$- \frac{1}{n} \log p(\mathbf{x}) \quad \text{(empirical entropy)}$$

$$= - \frac{1}{n} \log \prod_{k=1}^{n} p(x_k)$$

$$= - \frac{1}{n} \sum_{k=1}^{n} \log p(x_k)$$

$$= - \frac{1}{n} [N(0; \mathbf{x}) \log p(0) + N(1; \mathbf{x}) \log p(1) + N(2; \mathbf{x}) \log p(2)]$$

$$= - \frac{N(0; \mathbf{x})}{n} \log p(0) - \frac{N(1; \mathbf{x})}{n} \log p(1) - \frac{N(2; \mathbf{x})}{n} \log p(2)$$

$$= -q(0) \log 0.5 - q(1) \log 0.25 - q(2) \log 0.25$$

$$\approx \ H(X)$$

$$= -p(0) \log 0.5 - p(1) \log 0.25 - p(2) \log 0.25$$

$$= -(0.5) \log 0.5 - (0.25) \log 0.25 - (0.25) \log 0.25.$$  \hfill (1)

$$\approx \ H(X)$$

$$= -p(0) \log 0.5 - p(1) \log 0.25 - p(2) \log 0.25$$

$$= -(0.5) \log 0.5 - (0.25) \log 0.25 - (0.25) \log 0.25.$$  \hfill (2)
Weak Typicality $\not\Rightarrow$ Strong Typicality

1. Consider $X$ with distribution $p$ such that

$$p(0) = 0.5, \ p(1) = 0.25, \ p(2) = 0.25.$$

2. Consider a sequence $x$ of length $n$ and let

$$q(x) = n^{-1} N(x; x)$$

be the relative frequency of occurrence of symbol $x$ in $x$, $x = 0, 1, 2$.

3. In order for the sequence $x$ to be weakly typical, we need the empirical entropy to be close to $H(X)$:

$$-\frac{1}{n} \log p(x)$$

(3)

(1) and (2)

(4)

$$= -\frac{1}{n} \log \prod_{k=1}^{n} p(x_k)$$

(5)

$$= -\frac{1}{n} \sum_{k=1}^{n} \log p(x_k)$$

(6)

$$= -\frac{1}{n} [N(0; x) \log p(0) + N(1; x) \log p(1) + N(2; x) \log p(2)]$$

(7)

$$= -\frac{N(0; x)}{n} \log p(0) - \frac{N(1; x)}{n} \log p(1) - \frac{N(2; x)}{n} \log p(2)$$

(8)

$$= -q(0) \log 0.5 - q(1) \log 0.25 - q(2) \log 0.25$$

(9)

$$\approx H(X)$$

(10)

$$= -p(0) \log 0.5 - p(1) \log 0.25 - p(2) \log 0.25$$

(11)

$$= -(0.5) \log 0.5 - (0.25) \log 0.25 - (0.25) \log 0.25.$$
Weak Typicality $\nRightarrow$ Strong Typicality

1. Consider $X$ with distribution $p$ such that

\[ p(0) = 0.5, \; p(1) = 0.25, \; p(2) = 0.25. \]

2. Consider a sequence $x$ of length $n$ and let $q(x) = n^{-1} N(x; x)$ be the relative frequency of occurrence of symbol $x$ in $x$, $x = 0, 1, 2$.

3. In order for the sequence $x$ to be weakly typical, we need the empirical entropy to be close to $H(X)$:

\[
-\frac{1}{n} \log p(x) \quad \text{(empirical entropy)}
\]

\[
= -\frac{1}{n} \log \prod_{k=1}^{n} p(x_k)
\]

\[
= -\frac{1}{n} \sum_{k=1}^{n} \log p(x_k)
\]

\[
= -\frac{1}{n} \left[ N(0; x) \log p(0) + N(1; x) \log p(1) + N(2; x) \log p(2) \right]
\]

\[
= -\frac{N(0; x)}{n} \log p(0) - \frac{N(1; x)}{n} \log p(1) - \frac{N(2; x)}{n} \log p(2)
\]

\[
= -q(0) \log 0.5 - q(1) \log 0.25 - q(2) \log 0.25 \quad (1)
\]

\[
\approx H(X)
\]

\[
= -p(0) \log 0.5 - p(1) \log 0.25 - p(2) \log 0.25
\]

\[
= -(0.5) \log 0.5 - (0.25) \log 0.25 - (0.25) \log 0.25. \quad (2)
\]
Weak Typicality $\nRightarrow$ Strong Typicality

1. Consider $X$ with distribution $p$ such that

\[ p(0) = 0.5, \ p(1) = 0.25, \ p(2) = 0.25. \]

2. Consider a sequence $x$ of length $n$ and let

\[ q(x) = n^{-1} N(x; x) \]

be the relative frequency of occurrence of symbol $x$ in $x$, $x = 0, 1, 2$.

3. In order for the sequence $x$ to be weakly typical, we need the empirical entropy to be close to $H(X)$:

\[
\frac{1}{n} \log p(x) \quad \text{(empirical entropy)}
\]

\[
= \frac{1}{n} \log \prod_{k=1}^{n} p(x_k)
\]

\[
= \frac{1}{n} \sum_{k=1}^{n} \log p(x_k)
\]

\[
= \frac{1}{n} [N(0; x) \log p(0) + N(1; x) \log p(1) + N(2; x) \log p(2)]
\]

\[
= \frac{N(0; x)}{n} \log p(0) - \frac{N(1; x)}{n} \log p(1) - \frac{N(2; x)}{n} \log p(2)
\]

\[
= -q(0) \log 0.5 - q(1) \log 0.25 - q(2) \log 0.25 \quad (1)
\]

\[
\approx H(X)
\]

\[
= -p(0) \log 0.5 - p(1) \log 0.25 - p(2) \log 0.25
\]

\[
= -(0.5) \log 0.5 - (0.25) \log 0.25 - (0.25) \log 0.25. \quad (2)
\]
1. Consider $X$ with distribution $p$ such that

$$p(0) = 0.5, \ p(1) = 0.25, \ p(2) = 0.25.$$ 

2. Consider a sequence $x$ of length $n$ and let 
$q(x) = n^{-1} N(x; x)$ be the relative frequency of occurrence of symbol $x$ in $x$, $x = 0, 1, 2$.

3. In order for the sequence $x$ to be weakly typical, we need the empirical entropy to be close to $H(X)$:

$$- \frac{1}{n} \log p(x) \quad \text{(empirical entropy)}$$

$$= - \frac{1}{n} \log \prod_{k=1}^{n} p(x_k)$$

$$= - \frac{1}{n} \sum_{k=1}^{n} \log p(x_k)$$

$$= - \frac{1}{n} [N(0; x) \log p(0) + N(1; x) \log p(1) + N(2; x) \log p(2)]$$

$$= - \frac{N(0; x)}{n} \log p(0) - \frac{N(1; x)}{n} \log p(1) - \frac{N(2; x)}{n} \log p(2)$$

$$= - q(0) \log 0.5 - q(1) \log 0.25 - q(2) \log 0.25$$

$$\approx H(X)$$

$$= - p(0) \log 0.5 - p(1) \log 0.25 - p(2) \log 0.25$$

$$= -(0.5) \log 0.5 - (0.25) \log 0.25 - (0.25) \log 0.25. \quad (2)$$
Weak Typicality $\not\Rightarrow$ Strong Typicality

1. Consider $X$ with distribution $p$ such that

\[ p(0) = 0.5, \ p(1) = 0.25, \ p(2) = 0.25. \]

2. Consider a sequence $\mathbf{x}$ of length $n$ and let

\[ q(x) = n^{-1} N(x; \mathbf{x}) \] be the relative frequency of occurrence of symbol $x$ in $\mathbf{x}$, $x = 0, 1, 2$.

3. In order for the sequence $\mathbf{x}$ to be weakly typical, we need the empirical entropy to be close to $H(X)$:

\[ \frac{1}{n} \log p(\mathbf{x}) \quad (\text{empirical entropy}) \]

\[ = \frac{1}{n} \log \prod_{k=1}^{n} p(x_k) \]

\[ = \frac{1}{n} \sum_{k=1}^{n} \log p(x_k) \]

\[ = - \frac{1}{n} \left[ N(0; \mathbf{x}) \log p(0) + N(1; \mathbf{x}) \log p(1) + N(2; \mathbf{x}) \log p(2) \right] \]

\[ = - \frac{N(0; \mathbf{x})}{n} \log p(0) - \frac{N(1; \mathbf{x})}{n} \log p(1) - \frac{N(2; \mathbf{x})}{n} \log p(2) \]

\[ \approx - q(0) \log 0.5 - q(1) \log 0.25 - q(2) \log 0.25 \quad (1) \]

\[ \approx H(X) \]

\[ = - p(0) \log 0.5 - p(1) \log 0.25 - p(2) \log 0.25 \]

\[ = -(0.5) \log 0.5 - (0.25) \log 0.25 - (0.25) \log 0.25. \quad (2) \]
Weak Typicality \n\n1. Consider $X$ with distribution $p$ such that

$$p(0) = 0.5, \ p(1) = 0.25, \ p(2) = 0.25.$$ 

2. Consider a sequence $x$ of length $n$ and let

$$q(x) = n^{-1} N(x; x)$$

be the relative frequency of occurrence of symbol $x$ in $x$, $x = 0, 1, 2$.

3. In order for the sequence $x$ to be weakly typical, we need the empirical entropy to be close to $H(X)$:

$$- \frac{1}{n} \log p(x) \quad \text{(empirical entropy)}$$

$$= - \frac{1}{n} \log \prod_{k=1}^{n} p(x_k)$$

$$= - \frac{1}{n} \sum_{k=1}^{n} \log p(x_k)$$

$$= - \frac{1}{n} [N(0; x) \log p(0) + N(1; x) \log p(1) + N(2; x) \log p(2)]$$

$$= - \frac{N(0; x)}{n} \log p(0) - \frac{N(1; x)}{n} \log p(1) - \frac{N(2; x)}{n} \log p(2)$$

$$= -q(0) \log 0.5 - q(1) \log 0.25 - q(2) \log 0.25 \quad (1)$$

$$\approx H(X)$$

$$= -p(0) \log 0.5 - p(1) \log 0.25 - p(2) \log 0.25$$

$$= -(0.5) \log 0.5 - (0.25) \log 0.25 - (0.25) \log 0.25. \quad (2)$$
Weak Typicality $\not\Rightarrow$ Strong Typicality

1. Consider $X$ with distribution $p$ such that

$$p(0) = 0.5, \quad p(1) = 0.25, \quad p(2) = 0.25.$$ 

2. Consider a sequence $x$ of length $n$ and let $q(x) = n^{-1} N(x; x)$ be the relative frequency of occurrence of symbol $x$ in $x$, $x = 0, 1, 2$.

3. In order for the sequence $x$ to be weakly typical, we need the empirical entropy to be close to $H(X)$:

$$\frac{1}{n} \log p(x) \quad \text{(empirical entropy)}$$

$$= - \frac{1}{n} \log \prod_{k=1}^{n} p(x_k)$$

$$= - \frac{1}{n} \sum_{k=1}^{n} \log p(x_k)$$

$$= - \frac{1}{n} [N(0; x) \log p(0) + N(1; x) \log p(1) + N(2; x) \log p(2)]$$

$$= - \frac{N(0; x)}{n} \log p(0) - \frac{N(1; x)}{n} \log p(1) - \frac{N(2; x)}{n} \log p(2)$$

$$= - q(0) \log 0.5 - q(1) \log 0.25 - q(2) \log 0.25 \quad \text{(1)}$$

$$\approx H(X)$$

$$= - p(0) \log 0.5 - p(1) \log 0.25 - p(2) \log 0.25$$

$$= - (0.5) \log 0.5 - (0.25) \log 0.25 - (0.25) \log 0.25. \quad \text{(2)}$$
Weak Typicality $\not\Rightarrow$ Strong Typicality

1. Consider $X$ with distribution $p$ such that

$$p(0) = 0.5, \ p(1) = 0.25, \ p(2) = 0.25.$$ 

2. Consider a sequence $x$ of length $n$ and let

$$q(x) = n^{-1} N(x; x)$$

be the relative frequency of occurrence of symbol $x$ in $x$, $x = 0, 1, 2$.

3. In order for the sequence $x$ to be weakly typical, we need the empirical entropy to be close to $H(X)$:

$$-\frac{1}{n} \log p(x) \quad \text{(empirical entropy)}$$

$$= -\frac{1}{n} \log \prod_{k=1}^{n} p(x_k)$$

$$= -\frac{1}{n} \sum_{k=1}^{n} \log p(x_k)$$

$$= -\frac{1}{n} [N(0; x) \log p(0) + N(1; x) \log p(1) + N(2; x) \log p(2)]$$

$$= -\frac{N(0; x)}{n} \log p(0) - \frac{N(1; x)}{n} \log p(1) - \frac{N(2; x)}{n} \log p(2)$$

$$= -q(0) \log 0.5 - q(1) \log 0.25 - q(2) \log 0.25 \quad \quad (1)$$

$$\approx H(X)$$

$$= -p(0) \log 0.5 - p(1) \log 0.25 - p(2) \log 0.25$$

$$= -(0.5 \log 0.5 - 0.25 \log 0.25 - 0.25 \log 0.25). \quad \quad (2)$$

This can be satisfied by choosing $q(i) = p(i)$ for all $i$. 


Weak Typicality $\nRightarrow$ Strong Typicality

1. Consider $X$ with distribution $p$ such that

$$p(0) = 0.5, \; p(1) = 0.25, \; p(2) = 0.25.$$

2. Consider a sequence $x$ of length $n$ and let

$$q(x) = n^{-1} N(x; x)$$

be the relative frequency of occurrence of symbol $x$ in $x$, $x = 0, 1, 2$.

3. In order for the sequence $x$ to be weakly typical, we need the empirical entropy to be close to $H(X)$:

$$- \frac{1}{n} \log p(x) \quad \text{(empirical entropy)}$$

$$= - \frac{1}{n} \log \prod_{k=1}^{n} p(x_k)$$

$$= - \frac{1}{n} \sum_{k=1}^{n} \log p(x_k)$$

$$= - \frac{1}{n} [N(0; x) \log p(0) + N(1; x) \log p(1) + N(2; x) \log p(2)]$$

$$= - \frac{N(0; x)}{n} \log p(0) - \frac{N(1; x)}{n} \log p(1) - \frac{N(2; x)}{n} \log p(2)$$

$$= - q(0) \log 0.5 - q(1) \log 0.25 - q(2) \log 0.25$$

$$\approx H(X)$$

$$= - p(0) \log 0.5 - p(1) \log 0.25 - p(2) \log 0.25$$

$$= -(0.5) \log 0.5 - (0.25) \log 0.25 - (0.25) \log 0.25.$$

(1)

(2)

This can be satisfied by choosing $q(i) = p(i)$ for all $i$. 
Weak Typicality $\nRightarrow$ Strong Typicality

1. Consider $X$ with distribution $p$ such that
   
   \[ p(0) = 0.5, \quad p(1) = 0.25, \quad p(2) = 0.25. \]

2. Consider a sequence $\mathbf{x}$ of length $n$ and let
   \[ q(x) = n^{-1} N(x; \mathbf{x}) \]
   be the relative frequency of occurrence of symbol $x$ in $\mathbf{x}$, $x = 0, 1, 2$.

3. In order for the sequence $\mathbf{x}$ to be weakly typical, we need the empirical entropy to be close to $H(X)$:
   
   \[ \frac{1}{n} \log p(\mathbf{x}) \quad (\text{empirical entropy}) \]
   
   \[ = - \frac{1}{n} \log \prod_{k=1}^{n} p(x_k) \]
   
   \[ = - \frac{1}{n} \sum_{k=1}^{n} \log p(x_k) \]
   
   \[ = - \frac{1}{n} \left[ N(0; \mathbf{x}) \log p(0) + N(1; \mathbf{x}) \log p(1) + N(2; \mathbf{x}) \log p(2) \right] \]
   
   \[ = - \frac{N(0; \mathbf{x})}{n} \log p(0) - \frac{N(1; \mathbf{x})}{n} \log p(1) - \frac{N(2; \mathbf{x})}{n} \log p(2) \]

   \[ = -q(0) \log 0.5 - q(1) \log 0.25 - q(2) \log 0.25 \]
   
   \[ \approx H(\mathbf{X}) \]

   \[ = -p(0) \log 0.5 - p(1) \log 0.25 - p(2) \log 0.25 \]
   
   \[ = -(0.5) \log 0.5 - (0.25) \log 0.25 - (0.25) \log 0.25. \]

   \[ \text{This can be satisfied by choosing } q(i) = p(i) \text{ for all } i. \]
1. Consider $X$ with distribution $p$ such that

\[ p(0) = 0.5, \ p(1) = 0.25, \ p(2) = 0.25. \]

2. Consider a sequence $\mathbf{x}$ of length $n$ and let

\[ q(x) = n^{-1} N(x; \mathbf{x}) \]

be the relative frequency of occurrence of symbol $x$ in $\mathbf{x}$, $x = 0, 1, 2$.

3. In order for the sequence $\mathbf{x}$ to be weakly typical, we need the empirical entropy to be close to $H(X)$:

\[
\frac{1}{n} \log p(\mathbf{x}) \quad \text{(empirical entropy)}
\]

\[
= - \frac{1}{n} \log \prod_{k=1}^{n} p(x_k)
\]

\[
= - \frac{1}{n} \sum_{k=1}^{n} \log p(x_k)
\]

\[
= - \frac{1}{n} \left[ N(0; \mathbf{x}) \log p(0) + N(1; \mathbf{x}) \log p(1) + N(2; \mathbf{x}) \log p(2) \right]
\]

\[
= - \frac{N(0; \mathbf{x})}{n} \log p(0) - \frac{N(1; \mathbf{x})}{n} \log p(1) - \frac{N(2; \mathbf{x})}{n} \log p(2)
\]

\[
\approx -q(0) \log 0.5 - q(1) \log 0.25 - q(2) \log 0.25
\]

\[
= - p(0) \log 0.5 - p(1) \log 0.25 - p(2) \log 0.25
\]

\[
= - (0.5) \log 0.5 - (0.25) \log 0.25 - (0.25) \log 0.25.
\]

This can be satisfied by choosing $q(i) = p(i)$ for all $i$.
Weak Typicality $\nRightarrow$ Strong Typicality

1. Consider $X$ with distribution $p$ such that

$$p(0) = 0.5, \ p(1) = 0.25, \ p(2) = 0.25.$$  

2. Consider a sequence $\mathbf{x}$ of length $n$ and let

$$q(x) = n^{-1} N(x; \mathbf{x})$$

be the relative frequency of occurrence of symbol $x$ in $\mathbf{x}$, $x = 0, 1, 2$.

3. In order for the sequence $\mathbf{x}$ to be weakly typical, we need the empirical entropy to be close to $H(X)$:

$$-\frac{1}{n} \log p(\mathbf{x}) \quad \text{(empirical entropy)}$$

$$= -\frac{1}{n} \log \prod_{k=1}^{n} p(x_k)$$

$$= -\frac{1}{n} \sum_{k=1}^{n} \log p(x_k)$$

$$= -\frac{1}{n} \left[ N(0; \mathbf{x}) \log p(0) + N(1; \mathbf{x}) \log p(1) + N(2; \mathbf{x}) \log p(2) \right]$$

$$= -\frac{N(0; \mathbf{x})}{n} \log p(0) - \frac{N(1; \mathbf{x})}{n} \log p(1) - \frac{N(2; \mathbf{x})}{n} \log p(2)$$

$$= -q(0) \log 0.5 - q(1) \log 0.25 - q(2) \log 0.25$$

$$\approx H(X)$$

$$= -p(0) \log 0.5 - p(1) \log 0.25 - p(2) \log 0.25$$

$$= -(0.5) \log 0.5 - (0.25) \log 0.25 - (0.25) \log 0.25. \quad \text{(2)}$$

This can be satisfied by choosing $q(i) = p(i)$ for all $i$. 
Weak Typicality $\nRightarrow$ Strong Typicality

1. Consider $X$ with distribution $p$ such that

\[ p(0) = 0.5, \; p(1) = 0.25, \; p(2) = 0.25. \]

2. Consider a sequence $x$ of length $n$ and let

\[ q(x) = n^{-1} N(x; x) \]

be the relative frequency of occurrence of symbol $x$ in $x$, $x = 0, 1, 2$.

3. In order for the sequence $x$ to be weakly typical, we need the empirical entropy to be close to $H(X)$:

\[
- \frac{1}{n} \log p(x) \quad \text{(empirical entropy)}
\]

\[
= - \frac{1}{n} \log \prod_{k=1}^{n} p(x_k)
\]

\[
= - \frac{1}{n} \sum_{k=1}^{n} \log p(x_k)
\]

\[
= - \frac{1}{n} \left[ N(0; x) \log p(0) + N(1; x) \log p(1) + N(2; x) \log p(2) \right]
\]

\[
= - \frac{N(0; x)}{n} \log p(0) - \frac{N(1; x)}{n} \log p(1) - \frac{N(2; x)}{n} \log p(2)
\]

\[
= -q(0) \log 0.5 - q(1) \log 0.25 - q(2) \log 0.25 \quad \text{(1)}
\]

\[
\approx H(X)
\]

\[
= -p(0) \log 0.5 - p(1) \log 0.25 - p(2) \log 0.25
\]

\[
= -(0.5) \log 0.5 - (0.25) \log 0.25 - (0.25) \log 0.25. \quad \text{(2)}
\]

This can be satisfied by choosing $q(i) = p(i)$ for all $i$. 
Weak Typicality $\Rightarrow$ Strong Typicality

1. Consider $X$ with distribution $p$ such that

$$p(0) = 0.5, \quad p(1) = 0.25, \quad p(2) = 0.25.$$ 

2. Consider a sequence $x$ of length $n$ and let

$$q(x) = n^{-1} N(x; x)$$

be the relative frequency of occurrence of symbol $x$ in $x$, $x = 0, 1, 2$.

3. In order for the sequence $x$ to be weakly typical, we need the empirical entropy to be close to $H(X)$:

$$-\frac{1}{n} \log p(x) \quad \text{(empirical entropy)}$$

$$= -\frac{1}{n} \log \prod_{k=1}^{n} p(x_k)$$

$$= -\frac{1}{n} \sum_{k=1}^{n} \log p(x_k)$$

$$= -\frac{1}{n} [n(0; x) \log p(0) + n(1; x) \log p(1) + n(2; x) \log p(2)]$$

$$= -\frac{n(0; x)}{n} \log p(0) - \frac{n(1; x)}{n} \log p(1) - \frac{n(2; x)}{n} \log p(2)$$

$$= -q(0) \log 0.5 - q(1) \log 0.25 - q(2) \log 0.25$$

$$\approx H(X)$$

$$= -p(0) \log 0.5 - p(1) \log 0.25 - p(2) \log 0.25$$

$$= -(0.5) \log 0.5 - (0.25) \log 0.25 - (0.25) \log 0.25.$$  \hspace{1cm} (1)

This can be satisfied by choosing $q(i) = p(i)$ for all $i$. 

4. Alternatively, we can choose

$$q(0) = 0, \quad q(1) = 0, \quad q(2) = 0.$$ 

5. With such a choice of $\{q(i)\}$, the sequence $x$ is weakly typical with respect to $p$ because (1) and (2) are evaluated to the same value which implies empirical entropy $\approx H(X)$, but obviously not strongly typical with respect to $p$, because $p \neq q$. 

Weak Typicality $\nRightarrow$ Strong Typicality
Weak Typicality $\not\Rightarrow$ Strong Typicality

1. Consider $X$ with distribution $p$ such that

$$p(0) = 0.5, \; p(1) = 0.25, \; p(2) = 0.25.$$

2. Consider a sequence $\mathbf{x}$ of length $n$ and let

$$q(x) = n^{-1} N(x; \mathbf{x})$$

be the relative frequency of occurrence of symbol $x$ in $\mathbf{x}$, $x = 0, 1, 2$.

3. In order for the sequence $\mathbf{x}$ to be weakly typical, we need the empirical entropy to be close to $H(X)$:

$$-\frac{1}{n} \log p(\mathbf{x}) \quad (\text{empirical entropy})$$

$$= -\frac{1}{n} \log \prod_{k=1}^{n} p(x_k)$$

$$= -\frac{1}{n} \sum_{k=1}^{n} \log p(x_k)$$

$$= -\frac{1}{n} [N(0; \mathbf{x}) \log p(0) + N(1; \mathbf{x}) \log p(1) + N(2; \mathbf{x}) \log p(2)]$$

$$= -\frac{N(0; \mathbf{x})}{n} \log p(0) - \frac{N(1; \mathbf{x})}{n} \log p(1) - \frac{N(2; \mathbf{x})}{n} \log p(2)$$

$$= -q(0) \log 0.5 - q(1) \log 0.25 - q(2) \log 0.25$$

$$\approx H(X)$$

$$= -p(0) \log 0.5 - p(1) \log 0.25 - p(2) \log 0.25$$

$$= -(0.5) \log 0.5 - (0.25) \log 0.25 - (0.25) \log 0.25.$$ (1) (2)

This can be satisfied by choosing $q(i) = p(i)$ for all $i$. 

Weak Typicality $\nRightarrow$ Strong Typicality

1. Consider $X$ with distribution $p$ such that

\[ p(0) = 0.5, \quad p(1) = 0.25, \quad p(2) = 0.25. \]

2. Consider a sequence $x$ of length $n$ and let $q(x) = n^{-1}N(x; x)$ be the relative frequency of occurrence of symbol $x$ in $x$, $x = 0, 1, 2$.

3. In order for the sequence $x$ to be weakly typical, we need the empirical entropy to be close to $H(X)$:

\[ -\frac{1}{n} \log p(x) \quad \text{(empirical entropy)} \]

\[ = -\frac{1}{n} \log \prod_{k=1}^{n} p(x_k) \]

\[ = -\frac{1}{n} \sum_{k=1}^{n} \log p(x_k) \]

\[ = -\frac{1}{n} [N(0; x) \log p(0) + N(1; x) \log p(1) + N(2; x) \log p(2)] \]

\[ = -\frac{N(0; x)}{n} \log p(0) - \frac{N(1; x)}{n} \log p(1) - \frac{N(2; x)}{n} \log p(2) \]

\[ = -q(0) \log 0.5 - q(1) \log 0.25 - q(2) \log 0.25 \quad \text{(1)} \]

\[ \approx H(X) \]

\[ = -p(0) \log 0.5 - p(1) \log 0.25 - p(2) \log 0.25 \]

\[ = -(0.5) \log 0.5 - (0.25) \log 0.25 - (0.25) \log 0.25. \quad \text{(2)} \]

This can be satisfied by choosing $q(i) = p(i)$ for all $i$. 

1. Consider $X$ with distribution $p$ such that

$$p(0) = 0.5, \ p(1) = 0.25, \ p(2) = 0.25.$$  

2. Consider a sequence $x$ of length $n$ and let

$$q(x) = n^{-1} N(x; x)$$

be the relative frequency of occurrence of symbol $x$ in $x$, $x = 0, 1, 2$.

3. In order for the sequence $x$ to be weakly typical, we need the empirical entropy to be close to $H(X)$:

$$\frac{1}{n} \log p(x) \quad \text{(empirical entropy)}$$

$$= - \frac{1}{n} \log \prod_{k=1}^{n} p(x_k)$$

$$= - \frac{1}{n} \sum_{k=1}^{n} \log p(x_k)$$

$$= - \frac{1}{n} [N(0; x) \log p(0) + N(1; x) \log p(1) + N(2; x) \log p(2)]$$

$$= - \frac{N(0; x)}{n} \log p(0) - \frac{N(1; x)}{n} \log p(1) - \frac{N(2; x)}{n} \log p(2)$$

$$= - q(0) \log 0.5 - q(1) \log 0.25 - q(2) \log 0.25$$

$$\approx H(X)$$

$$= - p(0) \log 0.5 - p(1) \log 0.25 - p(2) \log 0.25$$

$$= - (0.5 \log 0.5 - 0.25 \log 0.25 - 0.25 \log 0.25).$$  

(1)

(2)

This can be satisfied by choosing $q(i) = p(i)$ for all $i$. 

---

**Weak Typicality $\iff$ Strong Typicality**

Weak Typicality $\neq$ Strong Typicality
Weak Typicality $\nRightarrow$ Strong Typicality

1. Consider $X$ with distribution $p$ such that

\[ p(0) = 0.5, \quad p(1) = 0.25, \quad p(2) = 0.25. \]

2. Consider a sequence $\mathbf{x}$ of length $n$ and let

\[ q(x) = n^{-1} N(x; \mathbf{x}) \]

be the relative frequency of occurrence of symbol $x$ in $\mathbf{x}$, $x = 0, 1, 2$.

3. In order for the sequence $\mathbf{x}$ to be weakly typical, we need the empirical entropy to be close to $H(X)$:

\[ -\frac{1}{n} \log p(\mathbf{x}) \] (empirical entropy)

\[
= -\frac{1}{n} \log \prod_{k=1}^{n} p(x_k) \\
= -\frac{1}{n} \sum_{k=1}^{n} \log p(x_k) \\
= -\frac{1}{n} \left[ N(0; \mathbf{x}) \log p(0) + N(1; \mathbf{x}) \log p(1) + N(2; \mathbf{x}) \log p(2) \right] \\
= -\frac{N(0; \mathbf{x})}{n} \log p(0) - \frac{N(1; \mathbf{x})}{n} \log p(1) - \frac{N(2; \mathbf{x})}{n} \log p(2) \\
= -q(0) \log 0.5 - q(1) \log 0.25 - q(2) \log 0.25 \\
\approx \quad H(X) \\
= -p(0) \log 0.5 - p(1) \log 0.25 - p(2) \log 0.25 \\
= - (0.5) \log 0.5 - (0.25) \log 0.25 - (0.25) \log 0.25. \]

This can be satisfied by choosing $q(i) = p(i)$ for all $i$. 
Weak Typicality ⇔ Strong Typicality

1. Consider $X$ with distribution $p$ such that

\[ p(0) = 0.5, \quad p(1) = 0.25, \quad p(2) = 0.25. \]

2. Consider a sequence $x$ of length $n$ and let $q(x) = n^{-1} N(x; x)$ be the relative frequency of occurrence of symbol $x$ in $x$, $x = 0, 1, 2$.

3. In order for the sequence $x$ to be weakly typical, we need the empirical entropy to be close to $H(X)$:

\[
-\frac{1}{n} \log p(x) \quad (\text{empirical entropy})
\]

\[
= -\frac{1}{n} \log \prod_{k=1}^{n} p(x_k)
\]

\[
= -\frac{1}{n} \sum_{k=1}^{n} \log p(x_k)
\]

\[
= -\frac{1}{n} [N(0; x) \log p(0) + N(1; x) \log p(1) + N(2; x) \log p(2)]
\]

\[
= -\frac{N(0; x)}{n} \log p(0) - \frac{N(1; x)}{n} \log p(1) - \frac{N(2; x)}{n} \log p(2)
\]

\[
= -q(0) \log 0.5 - q(1) \log 0.25 - q(2) \log 0.25 \quad (1)
\]

\[
\approx H(X)
\]

\[
= -p(0) \log 0.5 - p(1) \log 0.25 - p(2) \log 0.25
\]

\[
= -(0.5) \log 0.5 - (0.25) \log 0.25 - (0.25) \log 0.25. \quad (2)
\]

This can be satisfied by choosing $q(i) = p(i)$ for all $i$. 


Weak Typicality $\nRightarrow$ Strong Typicality

1. Consider $X$ with distribution $p$ such that
   
   $p(0) = 0.5, \quad p(1) = 0.25, \quad p(2) = 0.25$

2. Consider a sequence $\mathbf{x}$ of length $n$ and let $q(x) = n^{-1} N(x; \mathbf{x})$ be the relative frequency of occurrence of symbol $x$ in $\mathbf{x}$, $x = 0, 1, 2$.

3. In order for the sequence $\mathbf{x}$ to be weakly typical, we need the empirical entropy to be close to $H(X)$:

   \[
   - \frac{1}{n} \log p(\mathbf{x}) \quad \text{(empirical entropy)}
   \]

   \[
   = - \frac{1}{n} \log \prod_{k=1}^{n} p(x_k)
   \]

   \[
   = - \frac{1}{n} \sum_{k=1}^{n} \log p(x_k)
   \]

   \[
   = - \frac{1}{n} \left[ N(0; \mathbf{x}) \log p(0) + N(1; \mathbf{x}) \log p(1) + N(2; \mathbf{x}) \log p(2) \right]
   \]

   \[
   = - \frac{N(0; \mathbf{x})}{n} \log p(0) - \frac{N(1; \mathbf{x})}{n} \log p(1) - \frac{N(2; \mathbf{x})}{n} \log p(2)
   \]

   \[
   = -q(0) \log 0.5 - q(1) \log 0.25 - q(2) \log 0.25
   \]

   \[
   \approx H(X)
   \]

   \[
   = -p(0) \log 0.5 - p(1) \log 0.25 - p(2) \log 0.25
   \]

   \[
   = -(0.5) \log 0.5 - (0.25) \log 0.25 - (0.25) \log 0.25.
   \]

   (1)

   (2)

This can be satisfied by choosing $q(i) = p(i)$ for all $i$. 
1. Consider $X$ with distribution $p$ such that

$$p(0) = 0.5, \quad p(1) = 0.25, \quad p(2) = 0.25.$$ 

2. Consider a sequence $x$ of length $n$ and let $q(x) = n^{-1} N(x; x)$ be the relative frequency of occurrence of symbol $x$ in $x$, $x = 0, 1, 2$.

3. In order for the sequence $x$ to be weakly typical, we need the empirical entropy to be close to $H(X)$:

$$-\frac{1}{n} \log p(x) \quad \text{(empirical entropy)}$$

$$= -\frac{1}{n} \log \prod_{k=1}^{n} p(x_k)$$

$$= -\frac{1}{n} \sum_{k=1}^{n} \log p(x_k)$$

$$= -\frac{1}{n} \left[ N(0; x) \log p(0) + N(1; x) \log p(1) + N(2; x) \log p(2) \right]$$

$$= -\frac{N(0; x)}{n} \log p(0) - \frac{N(1; x)}{n} \log p(1) - \frac{N(2; x)}{n} \log p(2)$$

$$= -q(0) \log 0.5 - q(1) \log 0.25 - q(2) \log 0.25$$

$$\approx H(X)$$

$$= -p(0) \log 0.5 - p(1) \log 0.25 - p(2) \log 0.25$$

$$= -(0.5) \log 0.5 - (0.25) \log 0.25 - (0.25) \log 0.25. \quad (1)$$

This can be satisfied by choosing $q(i) = p(i)$ for all $i$. 

Weak Typicality $\nRightarrow$ Strong Typicality
1. Consider $X$ with distribution $p$ such that

\[ p(0) = 0.5, \quad p(1) = 0.25, \quad p(2) = 0.25. \]

2. Consider a sequence $\mathbf{x}$ of length $n$ and let

\[ q(x) = n^{-1} N(x; \mathbf{x}) \]

be the relative frequency of occurrence of symbol $x$ in $\mathbf{x}$, $x = 0, 1, 2$.

3. In order for the sequence $\mathbf{x}$ to be weakly typical, we need the empirical entropy to be close to $H(X)$:

\[
- \frac{1}{n} \log p(\mathbf{x}) \quad \text{(empirical entropy)}
\]

\[
= - \frac{1}{n} \log \prod_{k=1}^{n} p(x_k)
\]

\[
= - \frac{1}{n} \sum_{k=1}^{n} \log p(x_k)
\]

\[
= - \frac{1}{n} [N(0; \mathbf{x}) \log p(0) + N(1; \mathbf{x}) \log p(1) + N(2; \mathbf{x}) \log p(2)]
\]

\[
= - \frac{N(0; \mathbf{x})}{n} \log p(0) - \frac{N(1; \mathbf{x})}{n} \log p(1) - \frac{N(2; \mathbf{x})}{n} \log p(2)
\]

\[
= - q(0) \log 0.5 - q(1) \log 0.25 - q(2) \log 0.25 \quad \text{(1)}
\]

\[
\approx H(X)
\]

\[
= - p(0) \log 0.5 - p(1) \log 0.25 - p(2) \log 0.25
\]

\[
= -(0.5) \log 0.5 - (0.25) \log 0.25 - (0.25) \log 0.25. \quad \text{(2)}
\]

This can be satisfied by choosing $q(i) = p(i)$ for all $i$. 

**Weak Typicality $\nRightarrow$ Strong Typicality**
Weak Typicality $\not\Rightarrow$ Strong Typicality

1. Consider $X$ with distribution $p$ such that

$$ p(0) = 0.5, \quad p(1) = 0.25, \quad p(2) = 0.25 $$

2. Consider a sequence $x$ of length $n$ and let

$$ q(x) = n^{-1} N(x; x) $$

be the relative frequency of occurrence of symbol $x$ in $x$, $x = 0, 1, 2$.

3. In order for the sequence $x$ to be weakly typical, we need the empirical entropy to be close to $H(X)$:

$$ \frac{1}{n} \log p(x) \quad \text{(empirical entropy)} $$

$$ = -\frac{1}{n} \log \prod_{k=1}^{n} p(x_k) $$

$$ = -\frac{1}{n} \sum_{k=1}^{n} \log p(x_k) $$

$$ = -\frac{1}{n} [N(0; x) \log p(0) + N(1; x) \log p(1) + N(2; x) \log p(2)] $$

$$ = -\frac{N(0; x)}{n} \log p(0) - \frac{N(1; x)}{n} \log p(1) - \frac{N(2; x)}{n} \log p(2) $$

$$ = -q(0) \log 0.5 - q(1) \log 0.25 - q(2) \log 0.25 $$

$$ \approx H(X) $$

$$ = -p(0) \log 0.5 - p(1) \log 0.25 - p(2) \log 0.25 $$

$$ = -(0.5) \log 0.5 - (0.25) \log 0.25 - (0.25) \log 0.25 $$

(1)

(2)

This can be satisfied by choosing $q(i) = p(i)$ for all $i$. 
1. Consider $X$ with distribution $p$ such that
\[ p(0) = 0.5, \quad p(1) = 0.25, \quad p(2) = 0.25. \]

2. Consider a sequence $\mathbf{x}$ of length $n$ and let
\[ q(x) = n^{-1} N(x; \mathbf{x}) \]
be the relative frequency of occurrence of symbol $x$ in $\mathbf{x}$, $x = 0, 1, 2$.

3. In order for the sequence $\mathbf{x}$ to be weakly typical, we need the empirical entropy to be close to $H(X)$:
\[
- \frac{1}{n} \log p(x) \quad \text{(empirical entropy)}
\]
\[
= - \frac{1}{n} \log \prod_{k=1}^{n} p(x_k)
\]
\[
= - \frac{1}{n} \sum_{k=1}^{n} \log p(x_k)
\]
\[
= - \frac{1}{n} \left[ N(0; \mathbf{x}) \log p(0) + N(1; \mathbf{x}) \log p(1) + N(2; \mathbf{x}) \log p(2) \right]
\]
\[
= - \frac{N(0; \mathbf{x})}{n} \log p(0) - \frac{N(1; \mathbf{x})}{n} \log p(1) - \frac{N(2; \mathbf{x})}{n} \log p(2)
\]
\[
= - q(0) \log 0.5 - q(1) \log 0.25 - q(2) \log 0.25 \quad \text{(1)}
\]
\[
\approx H(X)
\]
\[
= - p(0) \log 0.5 - p(1) \log 0.25 - p(2) \log 0.25
\]
\[
= -(0.5) \log 0.5 - (0.25) \log 0.25 - (0.25) \log 0.25 \quad \text{(2)}
\]

This can be satisfied by choosing $q(i) = p(i)$ for all $i$. 

4. Alternatively, we can choose

\[ q(0) = 0.5, \quad q(1) = 0.25, \quad q(2) = 0.25. \]
1. Consider $X$ with distribution $p$ such that $p(0) = 0.5$, $p(1) = 0.25$, $p(2) = 0.25$.

2. Consider a sequence $x$ of length $n$ and let $q(x) = n^{-1} N(x; x)$ be the relative frequency of occurrence of symbol $x$ in $x$, $x = 0, 1, 2$.

3. In order for the sequence $x$ to be weakly typical, we need the empirical entropy to be close to $H(X)$:

$$\frac{1}{n} \log p(x) \quad \text{(empirical entropy)}$$

$$= \frac{1}{n} \log \prod_{k=1}^{n} p(x_k)$$

$$= \frac{1}{n} \log p(x_1) + \frac{1}{n} \log p(x_2) + \frac{1}{n} \log p(x_3)$$

$$= \frac{1}{n} [N(0; x) \log p(0) + N(1; x) \log p(1) + N(2; x) \log p(2)]$$

$$= \frac{N(0; x)}{n} \log p(0) - \frac{N(1; x)}{n} \log p(1) - \frac{N(2; x)}{n} \log p(2)$$

$$= -q(0) \log 0.5 - q(1) \log 0.25 - q(2) \log 0.25 \quad (1)$$

$$\approx H(X)$$

$$= -p(0) \log 0.5 - p(1) \log 0.25 - p(2) \log 0.25$$

$$= -(0.5) \log 0.5 - (0.25) \log 0.25 - (0.25) \log 0.25 \quad (2)$$

This can be satisfied by choosing $q(i) = p(i)$ for all $i$.

4. Alternatively, we can choose $q(0) = 0.5$, $q(1) = 0.5$, $q(2) = 0$. 

Weak Typicality $\not\Rightarrow$ Strong Typicality
1. Consider $X$ with distribution $p$ such that

\[ p(0) = 0.5, \quad p(1) = 0.25, \quad p(2) = 0.25. \]

2. Consider a sequence $\mathbf{x}$ of length $n$ and let

\[ q(x) = n^{-1} N(x; \mathbf{x}) \]

be the relative frequency of occurrence of symbol $x$ in $\mathbf{x}$, $x = 0, 1, 2$.

3. In order for the sequence $\mathbf{x}$ to be weakly typical, we need the empirical entropy to be close to $H(X)$:

\[
\frac{1}{n} \log p(\mathbf{x}) \quad (\text{empirical entropy})
\]

\[
= \frac{1}{n} \log \prod_{k=1}^{n} p(x_k)
\]

\[
= \frac{1}{n} \sum_{k=1}^{n} \log p(x_k)
\]

\[
= \frac{1}{n} \left[ N(0; \mathbf{x}) \log p(0) + N(1; \mathbf{x}) \log p(1) + N(2; \mathbf{x}) \log p(2) \right]
\]

\[
= -q(0) \log 0.5 - q(1) \log 0.25 - q(2) \log 0.25 \quad (1)
\]

\[
\approx H(X)
\]

\[
= -p(0) \log 0.5 - p(1) \log 0.25 - p(2) \log 0.25
\]

\[
= -(0.5) \log 0.5 - (0.25) \log 0.25 - (0.25) \log 0.25. \quad (2)
\]

This can be satisfied by choosing $q(i) = p(i)$ for all $i$.  

4. Alternatively, we can choose

\[ q(0) = 0.5, \quad q(1) = 0.5, \quad q(2) = 0. \]
1. Consider $X$ with distribution $p$ such that $p(0) = 0.5$, $p(1) = 0.25$, $p(2) = 0.25$.  

2. Consider a sequence $x$ of length $n$ and let $q(x) = n^{-1} N(x; x)$ be the relative frequency of occurrence of symbol $x$ in $x$, $x = 0, 1, 2$.  

3. In order for the sequence $x$ to be weakly typical, we need the empirical entropy to be close to $H(X)$:  

$$- \frac{1}{n} \log p(x) \quad \text{(empirical entropy)}$$  

$$= - \frac{1}{n} \log \prod_{k=1}^{n} p(x_k)$$  

$$= - \frac{1}{n} \sum_{k=1}^{n} \log p(x_k)$$  

$$= - \frac{1}{n} \left[ N(0; x) \log p(0) + N(1; x) \log p(1) + N(2; x) \log p(2) \right]$$  

$$= - \frac{N(0; x)}{n} \log p(0) - \frac{N(1; x)}{n} \log p(1) - \frac{N(2; x)}{n} \log p(2)$$  

$$= -q(0) \log 0.5 - q(1) \log 0.25 - q(2) \log 0.25$$ \hspace{1cm} (1)  

$$\approx H(X)$$  

$$= -p(0) \log 0.5 - p(1) \log 0.25 - p(2) \log 0.25$$  

$$= -[0.5 \log 0.5 - 0.25 \log 0.25 - 0.25 \log 0.25]$$ \hspace{1cm} (2)  

This can be satisfied by choosing $q(i) = p(i)$ for all $i$.  

4. Alternatively, we can choose $q(0) = 0.5$, $q(1) = 0.5$, $q(2) = 0$.  

Weak Typicality $\nRightarrow$ Strong Typicality
1. Consider $X$ with distribution $p$ such that $p(0) = 0.5$, $p(1) = 0.25$, $p(2) = 0.25$.

2. Consider a sequence $\mathbf{x}$ of length $n$ and let $q(x) = n^{-1}N(x; \mathbf{x})$ be the relative frequency of occurrence of symbol $x$ in $\mathbf{x}$, $x = 0, 1, 2$.

3. In order for the sequence $\mathbf{x}$ to be weakly typical, we need the empirical entropy to be close to $H(X)$:

$$
- \frac{1}{n} \log p(\mathbf{x}) \quad \text{(empirical entropy)}
$$

$$
= - \frac{1}{n} \log \prod_{k=1}^{n} p(x_k)
$$

$$
= - \frac{1}{n} \sum_{k=1}^{n} \log p(x_k)
$$

$$
= - \frac{1}{n} \left[ N(0; \mathbf{x}) \log p(0) + N(1; \mathbf{x}) \log p(1) + N(2; \mathbf{x}) \log p(2) \right]
$$

$$
= - \frac{N(0; \mathbf{x})}{n} \log p(0) - \frac{N(1; \mathbf{x})}{n} \log p(1) - \frac{N(2; \mathbf{x})}{n} \log p(2)
$$

$$
= - q(0) \log 0.5 - q(1) \log 0.25 - q(2) \log 0.25
$$

$$
\approx H(X)
$$

$$
= - p(0) \log 0.5 - p(1) \log 0.25 - p(2) \log 0.25
$$

$$
= - (0.5) \log 0.5 - (0.25) \log 0.25 - (0.25) \log 0.25
$$

(1) \quad (2)

This can be satisfied by choosing $q(i) = p(i)$ for all $i$. 

4. Alternatively, we can choose $q(0) = 0.5$, $q(1) = 0.5$, $q(2) = 0$. 

**Weak Typicality $\Rightarrow$ Strong Typicality**
1. Consider $X$ with distribution $p$ such that $p(0) = 0.5$, $p(1) = 0.25$, $p(2) = 0.25$. 

2. Consider a sequence $x$ of length $n$ and let $q(x) = n^{-1} N(x; x)$ be the relative frequency of occurrence of symbol $x$ in $x$, $x = 0, 1, 2$.

3. In order for the sequence $x$ to be weakly typical, we need the empirical entropy to be close to $H(X)$:

   \[
   \frac{1}{n} \log p(x) \quad \text{(empirical entropy)}
   \]

   \[
   = \frac{1}{n} \log \prod_{k=1}^{n} p(x_k)
   \]

   \[
   = \frac{1}{n} \sum_{k=1}^{n} \log p(x_k)
   \]

   \[
   = \frac{1}{n} \left[ N(0; x) \log p(0) + N(1; x) \log p(1) + N(2; x) \log p(2) \right]
   \]

   \[
   = - \frac{N(0; x)}{n} \log p(0) - \frac{N(1; x)}{n} \log p(1) - \frac{N(2; x)}{n} \log p(2)
   \]

   \[
   = -q(0) \log 0.5 - q(1) \log 0.25 - q(2) \log 0.25 \quad (1)
   \]

   \[
   \approx H(X)
   \]

   \[
   = -p(0) \log 0.5 - p(1) \log 0.25 - p(2) \log 0.25
   \]

   \[
   = -(0.5 \log 0.5 - 0.25 \log 0.25 - 0.25 \log 0.25) \quad (2)
   \]

   This can be satisfied by choosing $q(i) = p(i)$ for all $i$.

4. Alternatively, we can choose $q(0) = 0.5$, $q(1) = 0.5$, $q(2) = 0$. 

\[ Weak Typicality \not\Rightarrow Strong Typicality \]
1. Consider $X$ with distribution $p$ such that

\[ p(0) = 0.5, \quad p(1) = 0.25, \quad p(2) = 0.25. \]

2. Consider a sequence $\mathbf{x}$ of length $n$ and let

\[ q(x) = n^{-1} N(x; \mathbf{x}) \]

be the relative frequency of occurrence of symbol $x$ in $\mathbf{x}$, $x = 0, 1, 2$.

3. In order for the sequence $\mathbf{x}$ to be weakly typical, we need the empirical entropy to be close to $H(X)$:

\[
- \frac{1}{n} \log p(\mathbf{x}) \quad \text{(empirical entropy)}
\]

\[
= - \frac{1}{n} \log \prod_{k=1}^{n} p(x_k)
\]

\[
= - \frac{1}{n} \sum_{k=1}^{n} \log p(x_k)
\]

\[
= - \frac{1}{n} \left[ N(0; \mathbf{x}) \log p(0) + N(1; \mathbf{x}) \log p(1) + N(2; \mathbf{x}) \log p(2) \right]
\]

\[
= - \frac{N(0; \mathbf{x})}{n} \log p(0) - \frac{N(1; \mathbf{x})}{n} \log p(1) - \frac{N(2; \mathbf{x})}{n} \log p(2)
\]

\[
= - q(0) \log 0.5 - q(1) \log 0.25 - q(2) \log 0.25
\]

\[
\approx H(X)
\]

\[
= - p(0) \log 0.5 - p(1) \log 0.25 - p(2) \log 0.25
\]

\[
= - (0.5) \log 0.5 - (0.25) \log 0.25 - (0.25) \log 0.25
\]

This can be satisfied by choosing $q(i) = p(i)$ for all $i$. 4. Alternatively, we can choose

\[ q(0) = 0.5, \quad q(1) = 0.5, \quad q(2) = 0. \]
1. Consider \( X \) with distribution \( p \) such that
\[
p(0) = 0.5, \quad p(1) = 0.25, \quad p(2) = 0.25.
\]

2. Consider a sequence \( x \) of length \( n \) and let
\[
q(x) = n^{-1} N(x; x)\]
be the relative frequency of occurrence of symbol \( x \) in \( x, x = 0, 1, 2. \)

3. In order for the sequence \( x \) to be weakly typical, we need the empirical entropy to be close to \( H(X) \):
\[
-\frac{1}{n} \log p(x) \quad \text{(empirical entropy)}
\]
\[
= -\frac{1}{n} \log \prod_{k=1}^{n} p(x_k)
\]
\[
= -\frac{1}{n} \sum_{k=1}^{n} \log p(x_k)
\]
\[
= -\frac{1}{n} \left[ N(0; x) \log p(0) + N(1; x) \log p(1) + N(2; x) \log p(2) \right]
\]
\[
= -\frac{N(0; x)}{n} \log p(0) - \frac{N(1; x)}{n} \log p(1) - \frac{N(2; x)}{n} \log p(2)
\]
\[
= -q(0) \log 0.5 - q(1) \log 0.25 - q(2) \log 0.25 \quad \text{(1)}
\]
\[
= p(0) \log 0.5 - p(1) \log 0.25 - p(2) \log 0.25 \quad \text{(2)}
\]

This can be satisfied by choosing \( q(i) = p(i) \) for all \( i \).

4. Alternatively, we can choose
\[
q(0) = 0.5, \quad q(1) = 0.5, \quad q(2) = 0.
\]

5. With such a choice of \( \{q(i)\} \), the sequence \( x \) is weakly typical with respect to \( p \) because (1) and (2) are evaluated to the same value which implies

**Weak Typicality \( \nRightarrow \) Strong Typicality**

---

Weak Typicality  \( \nRightarrow \)  Strong Typicality
Weak Typicality $\nRightarrow$ Strong Typicality

1. Consider $X$ with distribution $p$ such that

\[ p(0) = 0.5, \quad p(1) = 0.25, \quad p(2) = 0.25. \]

2. Consider a sequence $x$ of length $n$ and let $q(x) = n^{-1} N(x; x)$ be the relative frequency of occurrence of symbol $x$ in $x$, $x = 0, 1, 2$.

3. In order for the sequence $x$ to be weakly typical, we need the empirical entropy to be close to $H(X)$:

\[
- \frac{1}{n} \log p(x) \quad \text{(empirical entropy)}
\]

\[
= - \frac{1}{n} \log \prod_{k=1}^{n} p(x_k)
\]

\[
= - \frac{1}{n} \sum_{k=1}^{n} \log p(x_k)
\]

\[
= - \frac{1}{n} \left[ N(0; x) \log p(0) + N(1; x) \log p(1) + N(2; x) \log p(2) \right]
\]

\[
= - \frac{N(0; x)}{n} \log p(0) - \frac{N(1; x)}{n} \log p(1) - \frac{N(2; x)}{n} \log p(2)
\]

\[
= - q(0) \log 0.5 - q(1) \log 0.25 - q(2) \log 0.25
\]

\[
\approx H(X)
\]

\[
= - (0.5) \log 0.5 - (0.25) \log 0.25 - (0.25) \log 0.25
\]

4. Alternatively, we can choose

\[ q(0) = 0.5, \quad q(1) = 0.5, \quad q(2) = 0. \]

5. With such a choice of \{q(i)\}, the sequence $x$ is weakly typical with respect to $p$ because (1) and (2) are evaluated to the same value which implies

\[ \text{empirical entropy} \approx H(X), \]

This can be satisfied by choosing $q(i) = p(i)$ for all $i$. 
Weak Typicality $\nRightarrow$ Strong Typicality

1. Consider $X$ with distribution $p$ such that

\[ p(0) = 0.5, \quad p(1) = 0.25, \quad p(2) = 0.25. \]

2. Consider a sequence $x$ of length $n$ and let $q(x) = n^{-1} N(x; x)$ be the relative frequency of occurrence of symbol $x$ in $x$, $x = 0, 1, 2$.

3. In order for the sequence $x$ to be weakly typical, we need the empirical entropy to be close to $H(X)$:

\[
\frac{1}{n} \log p(x) \quad \text{ (empirical entropy)}
\]

\[
= - \frac{1}{n} \log \prod_{k=1}^{n} p(x_k)
\]

\[
= - \frac{1}{n} \sum_{k=1}^{n} \log p(x_k)
\]

\[
= - \frac{1}{n} \left[ N(0; x) \log p(0) + N(1; x) \log p(1) + N(2; x) \log p(2) \right]
\]

\[
= - \frac{N(0; x)}{n} \log p(0) - \frac{N(1; x)}{n} \log p(1) - \frac{N(2; x)}{n} \log p(2)
\]

\[
\approx -q(0) \log 0.5 - q(1) \log 0.25 - q(2) \log 0.25 \quad \text{(1)}
\]

\[
\approx H(X)
\]

\[
= -p(0) \log 0.5 - p(1) \log 0.25 - p(2) \log 0.25
\]

\[
= -(0.5) \log 0.5 - (0.25) \log 0.25 - (0.25) \log 0.25 \quad \text{(2)}
\]

This can be satisfied by choosing $q(i) = p(i)$ for all $i$.

4. Alternatively, we can choose

\[ q(0) = 0.5, \quad q(1) = 0.5, \quad q(2) = 0. \]

5. With such a choice of $\{q(i)\}$, the sequence $x$ is weakly typical with respect to $p$ because (1) and (2) are evaluated to the same value which implies

\[ \text{empirical entropy} \approx H(X) , \]

but obviously not strongly typical with respect to $p$. 

\[ \text{Weak Typicality} \nRightarrow \text{Strong Typicality} \]
1. Consider $X$ with distribution $p$ such that

\[ p(0) = 0.5, \quad p(1) = 0.25, \quad p(2) = 0.25. \]

2. Consider a sequence $\mathbf{x}$ of length $n$ and let $q(\mathbf{x}) = n^{-1} N(x; \mathbf{x})$ be the relative frequency of occurrence of symbol $x$ in $\mathbf{x}$, $x = 0, 1, 2$.

3. In order for the sequence $\mathbf{x}$ to be weakly typical, we need the empirical entropy to be close to $H(X)$:

\[
-\frac{1}{n} \log p(\mathbf{x}) \quad \text{(empirical entropy)}
\]

\[
= -\frac{1}{n} \log \prod_{k=1}^{n} p(x_k)
\]

\[
= -\frac{1}{n} \sum_{k=1}^{n} \log p(x_k)
\]

\[
= -\frac{1}{n} \left[ N(0; \mathbf{x}) \log p(0) + N(1; \mathbf{x}) \log p(1) + N(2; \mathbf{x}) \log p(2) \right]
\]

\[
= - \frac{N(0; \mathbf{x})}{n} \log p(0) - \frac{N(1; \mathbf{x})}{n} \log p(1) - \frac{N(2; \mathbf{x})}{n} \log p(2)
\]

\[
= - q(0) \log 0.5 - q(1) \log 0.25 - q(2) \log 0.25
\]

\[
\approx H(X)
\]

\[
= - p(0) \log 0.5 - p(1) \log 0.25 - p(2) \log 0.25
\]

\[
= -(0.5) \log 0.5 - (0.25) \log 0.25 - (0.25) \log 0.25.
\]

This can be satisfied by choosing $q(i) = p(i)$ for all $i$.

4. Alternatively, we can choose

\[ q(0) = 0.5, \quad q(1) = 0.5, \quad q(2) = 0. \]

5. With such a choice of $\{q(i)\}$, the sequence $\mathbf{x}$ is weakly typical with respect to $p$ because (1) and (2) are evaluated to the same value which implies

\[
\text{empirical entropy} \approx H(X),
\]

but obviously not strongly typical with respect to $p$, because

\[ p \not\approx q. \]