DIRTY PAPER CODING AND DISTRIBUTED SOURCE CODING

TWO VIEWS OF COMBINED SOURCE AND CHANNEL CODING

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Summary

• INTRODUCTION

• DIRTY PAPER CODING
  - CODING FOR MEMORIES WITH DEFECTS
  - PARTITIONED LINEAR BLOCK CODES
  - COSET CODES

• DISTRIBUTED SOURCE CODING
  - BINNING VS QUANTIZATION
  - COSET CODES

• DISCUSSION
Information Theory
Some seminal papers by Shannon

• Channel Coding, 1948
• Source Coding, 1948, 1958
• Cryptography, 1949
Channel coding - example

Capacity: 1 bit/transmission
Best code: Use only inputs \{1, 3\}

Exercise moderation!!
Channel coding

Typically need larger codes, $n \gg 1$

Similar to sphere packing
Source coding:
Get good representation of source with few bits

\[ X^n \] source

\[ X^n \] Source encoder

\[ ^n X \] index

\[ ^n X \] Source decoder

Similar to sphere covering
**Channel Coding**

\[ C = \max_{p(x)} I(X;Y) \]

**Source Coding**

\[ R(D) = \min_{p(\hat{x}|x)} I(X;\hat{X}) \]

\[ p(\hat{x}|x): E d(x, \hat{x}) < D \]
Source and channel coding in communication system
Joint source and channel coding

Can be simple if source and channel are matched

Gaussian source

Gaussian noise

Power P

\[ X \rightarrow X \rightarrow + \rightarrow X \rightarrow \hat{X} \]
Rate distortion theory

Example: Gaussian source with memory

\[ D(R) = 2^{-2R} \sigma_x^2 \]

or

\[ R(D) = \frac{1}{2} \log_2 \left( \frac{\sigma_x^2}{D} \right) \]

\[ \therefore \text{Max } SNR(dB) = 10 \log_{10} \left( \frac{\sigma_x^2}{D(R)} \right) = 20R \log_{10} 2 \approx 6R \]
Dirty paper coding

CodiNG  foR  MeMoRies  wItH  DeFeCTs

1 0 0 1 1 1 0

“stuck-at” defects – probability $\alpha$

Binning: Randomly distribute all $2^n$ sequences into $2^{n_R}$ “bins”
# of sequences in each bin = \[ \frac{2^n}{2^{nR}} = 2 \]

\[ E \text{ (# of matching sequences in a bin)} = 2^{\frac{n(1-R)}{2}} \cdot 2^{-n\alpha} = 2^{n(1-\alpha-R)} \]
Note:

If $R < 1 - \alpha \rightarrow$ guaranteed to have a match

Thus Capacity = $1 - \alpha$ bits per memory cell

(same as if receiver knew defect positions)
C = \max \left( I(U;Y) - I(U;S) \right)

p(u,x|s)

In the example: \( U = Y \)
\[ C = \max \left( I(U;Y) - I(U;S) \right) \]

\[ \text{Adopt } U = X + \alpha S, \text{ maximize over } \alpha \]

Result:
\[ C = \frac{1}{2} \log \left( 1 + \frac{P}{N} \right), \text{ independently of } Q \]

Obtained with \( \alpha = \frac{P}{P+N} \)
Geometrical explanation:

\[ Y = X + S + Z \]

\[ U = X + \alpha S \]
Approximate methods

QIM (Quantization Index Modulation)

\[ f_\Delta (y) = \text{mod} \ (y+\Delta/2 \ , \ \Delta) - \Delta/2 \]

Encoding: \[ X = f_\Delta (U-S) = U - S - k \Delta, \quad k \text{ integer} \]

Decoding: \[ \hat{W} = f_\Delta (Y) = f_\Delta (U-S-k \Delta +S+Z) = f_\Delta (U + Z) \]
Watermark example

Images for watermark and host signal

Images for received watermark and received signal
Variations

- Partitioned Linear Block Codes (Heegard, 1983)
- Coset Codes (Forney, Ramchandran)
- Applications with BCH Codes, Reed-Solomon Codes
- Applications with Lattices
- Applications with LDPC, LDGM
Distributed source coding

\[
R(D) = \min_{p(w|x): \text{Ed}(X, \hat{X}) < D} \left( I(X; W) - I(Y; W) \right)
\]
Simple example

- X and Y vectors of size 3
- Hamming distance $\leq 1$

- **Case 1:** Y known by all (i.e., encoder and decoder)
  \[ R = H(X|Y) = 2 \text{ bits} \]  (just send X+Y)

- **Case 2:** Y known only by decoder – use coset codes
  Here too $R = 2 \text{ bits}$
  Send index of coset of X (use a repetition code)
  Decoding: Using coset of X and Y, recover X exactly
Repetition code – standard array

<table>
<thead>
<tr>
<th>Code (coset 0)</th>
<th>000</th>
<th>111</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coset 1</td>
<td>001</td>
<td>110</td>
</tr>
<tr>
<td>Coset 2</td>
<td>010</td>
<td>101</td>
</tr>
<tr>
<td>Coset 3</td>
<td>100</td>
<td>011</td>
</tr>
</tbody>
</table>

Can get X from Y and coset number
Another simple example:

Let $X$ and $Y$ be uniformly distributed length 7 binary sequences. The Hamming distance $(X,Y) \leq 1$

- $H(X) = H(Y) = 7$ bits
- $H(X|Y) = H(Y|X) = 3$ bits
- $H(X,Y) = 10$ bits

The achievable region is represented by the graph with $H(X)$ and $H(Y)$ values on the axes.
Encoding: use 3 bits (8 possible cosets)
   To encode X use coset of a Hamming (7,4) code

Decoding:
   Based on coset number and on Y, find X
Binning operation

Figure credit: K. Ramachandran
Dual operations

- **Quantization**
  integer division resulting in quotient

- **Binning**
  integer division resulting in remainder
Applications

- Digital Watermarking
- Steganography
- Cellular Telephony (Downlink)
- Cognitive Radio
- Radio Broadcasting
  - Digital-TV over Analog-TV (Chinook Comm., Boston Area)
  - Digital Radio over FM Radio (Alternative to IBOC and DRM)
- Video Compression (Distributed Source Coding)
- Video Synchronization
Information theory is alive and well!