



THE CHINESE UNIVERSITY OF HONG KONG  
 Institute of Network Coding  
 and  
 Department of Information Engineering  
*Seminar*



## Multicast Network Coding and Field Sizes

by

**Prof. Qifu (Tyler) Sun**  
**University of Science and Technology Beijing**

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**Time : 2:30 -3:30 pm**

**Venue : Room 833, Ho Sin Hang Engineering Building  
 The Chinese University of Hong Kong**

### Abstract

A fundamental result in network coding theory is that a multicast network is linearly solvable over every sufficiently large field. In particular, a linear solution over  $\text{GF}(q)$  can be efficiently constructed when  $q$  is no smaller than the number of receivers,.

In the first part of the talk, we reveal a surprising property that a multicast network linearly solvable over a given finite field is *not necessarily* linearly solvable over all larger finite fields, *i.e.*, it is possible to have  $q_{\min} < q_{\max}^*$ , where  $q_{\min}$  and  $q_{\max}^*$  stand for the minimum field size for the existence of a linear solution and the maximum field size for the non-existence of a linear solution, respectively. Specifically, we demonstrate three multicast networks each of which respectively has: (i)  $q_{\min} = 7$ ,  $q_{\max}^* = 8$ ; (ii)  $q_{\min} = 16$ ,  $q_{\max}^* = 17$ ; (iii)  $q_{\min} = 5$ ,  $q_{\max}^*$  equal to a Mersenne prime plus 1, which can be extremely large. The insight brought from these networks with the intriguing property  $q_{\min} < q_{\max}^*$  is that not only the field size  $q$ , but also the *order of multiplicative subgroups* in  $\text{GF}(q)$  plays an important role in the linear solvability over  $\text{GF}(q)$ .

In the second part of the talk, for every finite field pair ( $\text{GF}(q)$ ,  $\text{GF}(q')$ ) subject to a so-called *subgroup order criterion*, we propose a general framework to construct a multicast network linearly solvable over  $\text{GF}(q)$  but *not* over  $\text{GF}(q')$ . This general framework subsumes all exemplifying networks in the first part as special construction instances. Moreover, it can construct a series of new multicast networks with  $q_{\min} < q_{\max}^*$ . In particular, for any  $k \geq 2$ , an even more surprising network that is linearly solvable over  $\text{GF}(2^{2k})$  but *not* over  $\text{GF}(2^{2k+1})$  can be constructed. This reveals that the gap  $q_{\max}^* - q_{\min}$  can not only be positive, but tend to infinity as well.

### Biography

Qifu (Tyler) Sun received the B.Eng. (first class honors) and Ph.D. degrees from the Department of Information Engineering, The Chinese University of Hong Kong in 2005 and 2009, respectively. He has been a postdoctoral fellow at the Institute of Network Coding, The Chinese University of Hong Kong and a visiting fellow at the University of New South Wales. He is currently an associate professor at the School of Computer & Communication Engineering, University of Science and Technology Beijing. His research interests include fundamental study of network coding, channel coding and modulation, and algebraic study of communication networks.

**\*\* ALL ARE WELCOME \*\***