Constrained subgraph selection over coded packet networks

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Introduction to subgraph selection
- Minimum-cost subgraph selection with a single multicast session
- Constrained subgraph selection with a single multicast session
- Constrained subgraph selection with multiple multicast sessions
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Routing vs Network Coding

- Traditional routing in a node

- Network coding in a Node
Some benefits of network coding over routing

- Higher throughput
- Higher reliability
- Higher security
- Cheaper routing costs networks
- Lower delays
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**Network Model**

A network can be expressed as a directed graph $G = (N, A)$.

- $N$ denotes the set of nodes (routers or switches).
- $A$ denotes the set of directed arcs. Arcs represent the communication link between nodes.
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SESSIONS IN A NETWORK

Unicast transmission—One host sends and the other receives.

Broadcast transmission—One sender to all receivers.

Multicast transmission—One sender to a group of receivers.
Network coding can achieve the maximum multicast rate. It is not achievable by routing alone.

The problem of establishing multicast connection with network coding can be decomposed into two parts:

- Determining the subgraph to code over
- Determining the code to use over that subgraph
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DIFERENCE BETWEEN **Subgraph selection and coding**

- Subgraph selection and coding are very different problems!
  - Coding generally uses techniques from information theory and coding theory
  - Subgraph selection is essentially a problem of network resource allocation and generally uses techniques from networking theory

- **In this talk**, we focus to find an efficient subgraph that allows the given multicast connection to be established over coded packet networks

- **The analogous problem for routed network is the Steiner tree problem**, which is NP complete.
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We specify a multicast connection with a triplet \((s, T, R)\),

1. \(s\) is the source of the connection
2. \(T\) is the set of receivers
3. \(R\) is the multicast Rate

**Figure:** Butterfly network with multicast from \(s\) to \(t_1\) and \(t_2\).
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**VARIABLE AND PARAMETER NOTATIONS**

- **Variable notations**
  - $x_{ij}^{(t)}$ denotes the flow rate toward receiver $t$ on link $(i, j)$
  - $y_{ij}$ denotes the rate at which coded packets are injected onto link $(i, j)$

- **Parameter notations**
  - Cost per unit rate, $a_{ij}$
  - Capacity, $c_{ij}$

![Diagram](i,j)
### Variable and Parameter Notations

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RELATIONSHIP BETWEEN $x$ AND $z$

Subgraph definition:

The rate vector $z$, consisting of $z_{ij}, (i,j) \in A$, is called a subgraph.

$$z_{ij} = \max_{t \in T} (x_{ij}^{(t)}).$$
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Minimum-Cost Multicast Over Coded Packet Networks

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\min \sum_{(i,j) \in A} a_{ij} z_{ij}
\]

\[
s.t. z_{ij} = \max_{t \in T} (x_{ij}^{(t)}),
\]

\[
\sum_{\{j \mid (i,j) \in A\}} x_{ij}^{(t)} - \sum_{\{j \mid (j,i) \in A\}} x_{ji}^{(t)} = \begin{cases} R, & i = s; \\ -R, & i = t; \\ 0, & \text{otherwise} \end{cases}
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z_{ij} \leq c_{ij},
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\[z_{ij} \leq c_{ij}\]
Theorem:
There exists a network code flow arbitrarily close to $z_{ij}$ on each link $(i, j)$ for supporting a multicast connection of rate $R$ from source $s$ to $T$ if and only if the min-cut from $s$ to any $t \in T$ is greater than or equal to $R$, (Proof follows from min-cut max-flow).

This model can be solved in a
- Distributed way (using Lagrangian relaxation)
- Polynomial-time
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Summary
**What is QoS?**

- **Quality of Service (QoS)** is the capability of a network to provide better service.
- Without QoS, when you send some packet on the network, the packet can arrive in any order or take an undefined time to arrive.
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Various formal metrics to measure QoS

- **Delay**
  - The time taken by a packet to travel through the network from one end to another.

- **Delay Jitter**
  - The variation in the delay encountered by similar packets following the same route through the network.

- **Throughput**
  - The rate at which packets go through the network.

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  - The rate at which packets are dropped, get lost or become corrupted (some bits are changed in the packet) while going through the network.
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NC and QoS

- Nowadays, NC can be able to support multimedia applications like:
  - Video conferencing,
  - Audio conferencing,
  - FTP, HTTP service

- These real-time transactions are sensitive to network characteristics, such as delay, delay variation, bandwidth, and cost,
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Why do we need QoS?

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Consider a single session multicast in a network.

- Each link is marked with its cost per unit rate and weight.
- The weight could include delay, jitter, bandwidth, packet delivery ratio, and packet loss ratio.
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The problem is to find a subgraph over coded packet networks with

1. Minimum cost
2. Satisfying bandwidth constraints.
3. Longest end-to-end weight from the source to each destination does not exceed an upper bound.
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Let $P(k)$ denote the collection of all directed paths from source node $s$ to destination node $k$ in the underlying network $G$.

For example, we have three paths from $s$ to $t_1$: $P_1$ (Yellow one), $P_2$ (Green one), $P_3$ (Red one).

Define variable $f(p)$ as the flow on path $p \in P(k)$. 

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**Path-based formulation**

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The weight of path $p \in P^{(k)}$ is defined as follows:

$$W^{(k)}(p) = \sum_{e \in p} w_e. \quad (1)$$

The following constraint is considered to guarantee the longest end-to-end violation.

$$\max_{p \in P^{(k)}} (W^{(k)}(p)) \leq U^{(k)}, \quad \forall k \in K. \quad (2)$$

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The amount of a link flow, \( x_e^{(k)} \), is computed from path flows by the following relation.

\[
x_e^{(k)} = \sum_{p \in P^{(k)}} \delta_e(p)f(p)
\]

For example \( x_{6t1}^{(1)} \) is equal to \( f(2) + f(3) \).

The rate at which coded packets are injected onto link \( e \).

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z_e = \max_{k \in K} \left( \sum_{p \in P^{(k)}} \delta_e(p)f(p) \right).
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Path-based formulation for the problem of finding an constrained multicast sub-graph

\[
\min \sum_{e \in E} c_e z_e
\]

s.t. \[\sum_{p \in P^{(k)}} f(p) = R, \quad \forall k \in K,\]

\[z_e = \max \left( \sum_{k \in K} \delta_e(p) f(p) \right), \quad \forall e \in E,\]

\[0 \leq z_e \leq u_e, \quad \forall e \in E,\]

\[\max_{p \in P^{(k)}} (W^{(k)}(p)) \leq U^{(k)}, \quad \forall k \in K.\]

\[W^{(k)}(p) = \sum_{e \in p} w_e, \quad \forall p \in P^{(k)},\]

\[0 \leq f(p), \quad \forall p \in P^{(k)}.\]

- Minimizes total cost
- Flow conservation
- Coded packet Rate
- Capacity constraint
- End-to-end weight
Path-based formulation for the problem of finding an constrained multicast sub-graph

\[
\begin{align*}
\min & \quad \sum_{e \in E} c_e z_e \\
\text{s.t.} & \quad \sum_{p \in P(k)} f(p) = R, \quad \forall k \in K, \\
& \quad z_e = \max_{k \in K} (\sum_{p \in P(k)} \delta_e(p) f(p)), \quad \forall e \in E, \\
& \quad 0 \leq z_e \leq u_e, \quad \forall e \in E, \\
& \quad \max_{p \in P(k)} (W^{(k)}(p)) \leq U^{(k)}, \quad \forall k \in K. \\
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& \quad 0 \leq f(p), \quad \forall p \in P^{(k)}. 
\end{align*}
\]

- Minimizes total cost
- Flow conservation
- Coded packet Rate
- Capacity constraint
  \{ End-to-end weight \}
Path-based formulation for the problem of finding an constrained multicast sub-graph

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- Minimizes total cost
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**End-to-end weight**
Path-based formulation for the problem of finding an
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\[
\begin{align*}
\min & \quad \sum_{e \in E} c_e z_e \\
\text{s.t.} & \quad \sum_{p \in \mathcal{P}(k)} f(p) = R, \quad \forall k \in K, \\
& \quad z_e = \max (\sum_{p \in \mathcal{P}(k)} \delta_e(p)f(p)), \quad \forall e \in E, \\
& \quad 0 \leq z_e \leq u_e, \quad \forall e \in E, \\
& \quad \max_{p \in \mathcal{P}(k)} (\mathcal{W}(k)(p)) \leq U(k), \quad \forall k \in K, \\
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- Minimizes total cost
- Flow conservation
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This model can be converted into a mixed-integer linear programming problem.

The problem is also NP-hard. Because a constrained shortest path problem can be reduced to it.

The problem can be solved in a distributed method.

The proposed algorithm includes:

- Column generation method to find upper bounds on the optimum objective value
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Summary
Stochastic Delay

- Delay is one of the most important QoS parameters for real time services,
- In a single multicast session, the delay usually assume a fixed deterministic value.
- In multiple multicast sessions, the delay usually assumed to be stochastic.
Stochastic Delay

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Delay is one of the most important QoS parameters for real time services,

In a single multicast session, the delay usually assume a fixed deterministic value.

In multiple multicast sessions, the delay usually assumed to be stochastic.
Each session $m \in M$ is identified by the source-destination pair $(s_m, T_m, R_m)$,

1. $s_m$ is the source node
2. $T_m$ is the set of receivers of session $m$
3. $R_m$ is multicast rate

- : Camera sensor  - : Sink Node
Each session $m \in M$ is identified by the source-destination pair $(s_m, T_m, R_m)$,

1. $s_m$ is the source node
2. $T_m$ is the set of receivers of session $m$.
3. $R_m$ is multicast rate.
Assume that the random variable $d_e$ is characterized by:
1. Mean, $\bar{d}_e$, 
2. Variance, $\sigma^2_e$

Let $P_{m,k}$ denote the collection of all directed paths from source node, $s^m$, to destination node, $k$, in session $m$.

The end-to-end statistical delay of path $p \in P_{m,k}$ is defined as follows:

$$D_{m,k}(p) = \sum_{e \in p} d_e.$$ (3)
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**STATISTICAL DELAY CONSTRAINTS**

- $D_{m,k}^{max}$ denotes the maximum tolerable delay,
- $\beta_{m,k}^{max}$ denotes the violation probability of the delay constraint from source node, $s^m$, to destination node, $k$, in session $m$,

\[
\Pr(D_{m,k}^{max}(p) \leq D_{m,k}^{max}) = 1 - \beta_{m,k}^{max}. \tag{4}
\]
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$$Pr(D_{m,k}(p) \leq D_{max}^{m,k}) = 1 - \beta_{m,k}.$$  (4)
Using Markov’s inequality, we have:

\[ Pr(D_{m,k}^m(p) \geq D_{m,k}^{m,k}) \leq \frac{E(D_{m,k}^m(p))}{D_{m,k}^{m,k}}, \quad (5) \]

- \( E(D_{m,k}^m(p)) = \sum_{e \in p} \bar{d}_e \).
- Hence, \( \text{Delay}(p) \) for path \( p \in P_{m,k}^m \) is defined as follows:
  \[ \text{Delay}(p) = \begin{cases} \frac{\sum_{e \in p} \bar{d}_e}{D_{m,k}^{m,k}}, & \text{if } f(p) > 0, \\ 0, & \text{Otherwise.} \end{cases} \]
Using Markov’s inequality, we have:

\[ Pr(D_{m,k}^m(p) \geq D_{m,k}^{m,k}) \leq \frac{E(D_{m,k}^m(p))}{D_{m,k}^{m,k}} \]  

\[ E(D_{m,k}^m(p)) = \sum_{e \in p} \bar{d}_e. \]

Hence, Delay(p) for path \( p \in P^{m,k} \) is defined as follows:

\[ \begin{align*}
\text{Delay}(p) &= \left\{ \begin{array}{ll}
\frac{\sum_{e \in p} \bar{d}_e}{D_{m,k}^{m,k}}, & \text{if } f(p) > 0, \\
0, & \text{Otherwise.}
\end{array} \right.
\]
Using Markov’s inequality, we have:

\[
Pr(D_{m,k}^m(p) \geq D_{m,k}^{\text{max}}) \leq \frac{E(D_{m,k}^m(p))}{D_{m,k}^{\text{max}}} \tag{5}
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**Bounds on end-to-end jitter constraints**

- **Jitter** can be defined as the maximum difference between the real-time packet delay and mean delay computed empirically.

![Diagram showing jitter constraints](image)
The probability that the path, \( p \in P_{m,k} \), satisfies the jitter constraint is

\[
Pr\left( |D_{m,k}(p) - E(D_{m,k}(p))| \leq J_{m,k} \right) = 1 - \alpha_{m,k}
\]

Using Tchebichev’s inequality, we have

\[
Pr\left( |D_{m,k}(p) - E(D_{m,k}(p))| \geq J_{m,k} \right) \leq \frac{V(D_{m,k}(p))}{(J_{m,k})^2}
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where \( V(D_{m,k}(p)) \) is the end-to-end delay’s variance.
The probability that the path, \( p \in P^{m,k} \), satisfies the jitter constraint is

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$$\Pr(|D_{m,k}^{m,k}(p) - E(D_{m,k}^{m,k}(p))| \leq J_{m,k}) = 1 - \alpha_{m,k}$$

Using Tchebitchev’s inequality, we have

$$\Pr(|D_{m,k}^{m,k}(p) - E(D_{m,k}^{m,k}(p))| \geq J_{m,k}) \leq \frac{V(D_{m,k}^{m,k}(p))}{(J_{m,k})^2}$$

where $V(D_{m,k}^{m,k}(p))$ is the end-to-end delay’s variance.
With assuming independent delays for each link, we have

$$V(D_{m,k}^m(p)) = \sum_{e \in P} \sigma_e^2$$

Jitter($p$) for path $p \in P_{m,k}^m$ is defined as follows:

$$\text{Jitter}(p) = \begin{cases} \frac{\sum_{e \in P} \sigma_e^2}{(J_{m,k})^2}, & \text{if } f(p) > 0, \\ 0, & \text{otherwise}. \end{cases}$$
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RELATIONSHIP BETWEEN $x$ AND $z$

- Then, the link flow, $x_{e}^{m,k}$, can be written into the path flows as follows:

$$x_{e}^{m,k} = \sum_{p \in P^{m,k}} \delta_{e}^{m,k}(p)f(p). \quad (6)$$

- Coded packet rate injected on link $e$ for session $m$ is as follows:

$$z_{e}^{m} = \max_{k \in T^{m}} \left( \sum_{p \in P^{m,k}} \delta_{e}^{m,k}(p)f(p) \right),$$
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**Path-based formulation**

\[
\begin{align*}
\min & \quad \sum_{e \in A} \sum_{m \in M} c_e \max_k \left( \sum_{p \in P_{m,k}} \delta_{e}^{m,k}(p)f(p) \right) \\
\text{s.t.} & \quad \sum_{p \in P_{m,k}} f(p) = R^m, \\
& \quad z_e^m = \max_k \left( \sum_{p \in P_{m,k}} \delta_{e}^{m,k}(p)f(p) \right), \\
& \quad 0 \leq \sum_{m \in M} z_e^m \leq u_e, \\
& \quad \max_{p \in P_{m,k}} \{\text{Delay}(p)\} \leq \beta_{m,k}, \\
& \quad \max_{p \in P_{m,k}} \{\text{Jitter}(p)\} \leq \alpha_{m,k},
\end{align*}
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- Minimizes the total cost
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**Path-based formulation**

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\begin{align*}
\text{min} & \quad \sum_{e \in A} \sum_{m \in M} c_e \max_{k \in T^m} \left( \sum_{p \in P_{m,k}} \delta_{e,k}^m(p) f(p) \right) \\
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& \quad \max_{p \in P^{m,k}} \{\text{Jitter}(p)\} \leq \alpha^{m,k}, \\
& \quad f(p) \geq 0.
\end{align*}
\]

- Minimizes the total cost
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**Path-based formulation**

Minimizes the total cost
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\begin{align*}
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\end{align*}
\]

- Minimizes the total cost
- Flow conservation constraint
- Coded packet rate
- Capacity constraint
- Delay constraints
- Jitter constraints
Minimizes the total cost

Flow conservation constraint

Coded packet rate

Capacity constraint

Delay constraints

Jitter constraints
The Model can be rewritten as a mixed-integer linear programming

- The problem is NP-hard. Because, a two-constraint knapsack problem can reduce to it.
- The proposed algorithm is based on a primal and dual decomposition methods.
  - Primal decomposition method provides an upper bound of the objective value.
  - Dual decomposition method provides a lower bound of the objective value.
The Model can be rewritten as a mixed-integer linear programming

The problem is NP-hard. Because, a two-constraint knapsack problem can reduce to it.

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