5.2 The Source Coding Theorem
The encoder maps a random source sequence $X$ to an index $f(X)$ in an index set $I = \{1, 2, \ldots, M\}$. Such a code is called a block code with $n$ being the block length of the code. The encoder sends $f(X)$ to the decoder through a noiseless channel. Based on the index, the decoder outputs $\hat{X}$ as an estimate on $X$. 
The encoder maps a random source sequence $\mathbf{X} \in \mathcal{X}^n$ to an index $f(\mathbf{X})$ in an index set

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- The encoder maps a random source sequence $X \in \mathcal{X}^n$ to an index $f(X)$ in an index set

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- The encoder sends $f(\mathbf{X})$ to the decoder through a noiseless channel.

- Based on the index, the decoder outputs $\hat{\mathbf{X}}$ as an estimate on $\mathbf{X}$. 
• The encoder is specified by the mapping

\[ f : \mathcal{X}^n \rightarrow \mathcal{I} = \{1, 2, \cdots, M\}. \]
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• The rate of the code is given by \( R = n^{-1} \log M \) in bits per source symbol, where \( M \) is the size of the index set and \( n \) is the block length.
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If \( M = |\mathcal{X}^n| = |\mathcal{X}|^n \), the rate of the code is

\[ \frac{1}{n} \log M = \frac{1}{n} \log |\mathcal{X}|^n = \log |\mathcal{X}|. \]
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• Typically, \( R < \log |\mathcal{X}| \) for data compression.
The encoder is specified by the mapping

$$f : \mathcal{X}^n \rightarrow \mathcal{I} = \{1, 2, \cdots, M\}.$$  

The rate of the code is given by $R = n^{-1} \log M$ in bits per source symbol, where $M$ is the size of the index set and $n$ is the block length.

If $M = |\mathcal{X}^n| = |\mathcal{X}|^n$, the rate of the code is

$$\frac{1}{n} \log M = \frac{1}{n} \log |\mathcal{X}|^n = \log |\mathcal{X}|.$$  

Typically, $R < \log |\mathcal{X}|$ for data compression.

An error occurs if $\hat{X} \neq X$, and $P_e = \Pr\{\hat{X} \neq X\}$ is called the error probability.
The Source Coding Theorem

**Direct Part:** For arbitrarily small $P_e$, there exists a block code whose coding rate is arbitrarily close to $H(X)$ when $n$ is sufficiently large.
The Source Coding Theorem

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- This part says that reliable communication can be achieved if the coding rate is at least $H(X)$. 
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**Converse:** For any block code with block length $n$ and coding rate less than $H(X) - \zeta$, where $\zeta > 0$ does not change with $n$, then $P_e \to 1$ as $n \to \infty$. 
The Source Coding Theorem

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- This part says that reliable communication can be achieved if the coding rate is at least $H(X)$.

**Converse:** For any block code with block length $n$ and coding rate less than $H(X) - \zeta$, where $\zeta > 0$ does not change with $n$, then $P_e \to 1$ as $n \to \infty$.

- This part says that it is impossible to achieve reliable communication if the coding rate is less than $H(X)$. 


Direct Part

• For an arbitrarily small but fixed $\epsilon > 0$, construct a sequence of codes with block length $n$ such that $P_e < \epsilon$ when $n$ is sufficiently large.
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• We will consider a class of block codes with a particular structure.
A Class of Block Codes
A Class of Block Codes

Encoder:

1. Choose a subset $\mathcal{A}$ of $X^n$ and let $M = |\mathcal{A}|$. 

Decoder:

1. For an index $i \in I$, decode it to the unique $x \in \mathcal{A}$ such that $f(x) = i$.
2. If the source sequence $x \in \mathcal{A}$, then it is decoded correctly.
3. If source sequence $x \notin \mathcal{A}$, then it is decoded incorrectly.
4. Thus $P_e = Pr\{X \notin \mathcal{A}\}$.
Encoder:
1. Choose a subset $A$ of $X^n$ and let $M = |A|$.
2. For each source sequence $x \in A$, assign it to a unique index $f(x) \in I$, i.e.,
   \[ f(x) \neq f(x') \text{ for } x \neq x'. \]
A Class of Block Codes

Encoder:
1. Choose a subset $\mathcal{A}$ of $\mathcal{X}^n$ and let $M = |\mathcal{A}|$.
2. For each source sequence $\mathbf{x} \in \mathcal{A}$, assign it to a unique index $f(\mathbf{x}) \in \mathcal{I}$, i.e.,
   \[ f(\mathbf{x}) \neq f(\mathbf{x}') \quad \text{for} \quad \mathbf{x} \neq \mathbf{x}' . \]
3. For each source sequence $\mathbf{x} \not\in \mathcal{A}$, let $f(\mathbf{x}) = 1$.

Decoder:
1. For an index $i \in \mathcal{I}$, decode it to the unique $\mathbf{x} \in \mathcal{A}$ such that $f(\mathbf{x}) = i$.
2. If the source sequence $\mathbf{x} \in \mathcal{A}$, then it is decoded correctly.
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$$
\begin{array}{c}
\mathcal{X}^n \\
\mathcal{A} \\
\mathcal{I} \\
1 \\
2 \\
\vdots \\
M \\
\end{array}
$$
A Class of Block Codes

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\[ \mathcal{X}^n \quad \mathcal{A} \quad \mathcal{I} \quad \begin{array}{c} 1 \\ 2 \\ \vdots \\ M \end{array} \]
Source Coding Theorem (Direct Part)
For arbitrarily small $P_e$, there exists a block code whose coding rate is arbitrarily close to $H(X)$ when $n$ is sufficiently large.

Proof

**Theorem 5.2 (Weak AEP II)**

1) If $x \in W^n_{[X]_\epsilon}$, then

$$2^{-n(H(X)+\epsilon)} \leq p(x) \leq 2^{-n(H(X)-\epsilon)}.$$  

2) For $n$ sufficiently large,

$$\Pr\{X \notin W^n_{[X]_\epsilon}\} > 1 - \epsilon.$$  

3) For $n$ sufficiently large,

$$(1 - \epsilon)2^{-n(H(X) - \epsilon)} \leq |W^n_{[X]_\epsilon}| \leq 2^n(H(X) + \epsilon).$$
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5. By WAEP,
\[P_e = \Pr\{X \not\in \mathcal{A}\} = \Pr\{X \not\in W^n_{[X]\epsilon}\} < \epsilon.
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---

**Theorem 5.2 (Weak AEP II)**

1. If $x \in W_{[X]}^n\varepsilon$, then

   \[2^{-n(H(X) + \varepsilon)} \leq p(x) \leq 2^{-n(H(X) - \varepsilon)}.\]

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   \[\Pr\{X \in W_{[X]}^n\varepsilon\} > 1 - \varepsilon.\]

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Proof
1. We need to choose the set $\mathcal{A}$ suitably.
2. Fix $\epsilon > 0$ and take $\mathcal{A} = W^n_{[X]^\epsilon}$ and hence $M = |\mathcal{A}|$.
3. For sufficiently large $n$, by WAEP,
   $$(1-\epsilon)2^{n(H(X)-\epsilon)} \leq M = |\mathcal{A}| = |W^n_{[X]^\epsilon}| \leq 2^n(H(X)+\epsilon).$$
4. The coding rate satisfies
   $$\frac{1}{n} \log(1 - \epsilon) + H(X) - \epsilon \leq \frac{1}{n} \log M \leq H(X) + \epsilon.$$
5. By WAEP,
   $$P_e = \Pr\{X \not\in \mathcal{A}\} = \Pr\{X \not\in W^n_{[X]^\epsilon}\} < \epsilon.$$
6. Letting $\epsilon \to 0$, the coding rate tends to $H(X)$, while $P_e$ tends to 0.

Theorem 5.2 (Weak AEP II)
1) If $x \in W^n_{[X]^\epsilon}$, then
   $$2^{-n(H(X)+\epsilon)} \leq p(x) \leq 2^{-n(H(X)-\epsilon)}.$$
2) For $n$ sufficiently large,
   $$\Pr\{X \in W^n_{[X]^\epsilon}\} > 1 - \epsilon.$$
3) For $n$ sufficiently large,
   $$(1 - \epsilon)2^{n(H(X)-\epsilon)} \leq |W^n_{[X]^\epsilon}| \leq 2^{n(H(X)+\epsilon)}.$$
Source Coding Theorem (Direct Part)
For arbitrarily small $P_e$, there exists a block code whose coding rate is arbitrarily close to $H(X)$ when $n$ is sufficiently large.

Proof
1. We need to choose the set $\mathcal{A}$ suitably.
2. Fix $\epsilon > 0$ and take $\mathcal{A} = W^n_{[X]\epsilon}$ and hence $M = |\mathcal{A}|$.
3. For sufficiently large $n$, by WAEP,

\[(1 - \epsilon)2^n(H(X) - \epsilon) \leq M = |\mathcal{A}| = |W^n_{[X]\epsilon}| \leq 2^n(H(X) + \epsilon).
\]
4. The coding rate satisfies

\[
\frac{1}{n} \log(1 - \epsilon) + H(X) - \epsilon \leq \frac{1}{n} \log M \leq H(X) + \epsilon.
\]

5. By WAEP,

\[P_e = \Pr\{X \not\in \mathcal{A}\} = \Pr\{X \not\in W^n_{[X]\epsilon}\} < \epsilon.
\]
6. Letting $\epsilon \to 0$, the coding rate tends to $H(X)$, while $P_e$ tends to 0.

Theorem 5.2 (Weak AEP II)

1) If $x \in W^n_{[X]\epsilon}$, then

\[2^{-n(H(X) + \epsilon)} \leq p(x) \leq 2^{-n(H(X) - \epsilon)}.
\]

2) For $n$ sufficiently large,

\[\Pr\{X \in W^n_{[X]\epsilon}\} > 1 - \epsilon.
\]

3) For $n$ sufficiently large,

\[(1 - \epsilon)2^n(H(X) - \epsilon) \leq |W^n_{[X]\epsilon}| \leq 2^n(H(X) + \epsilon).
\]
**Source Coding Theorem (Direct Part)**

For arbitrarily small $P_e$, there exists a block code whose coding rate is arbitrarily close to $H(X)$ when $n$ is sufficiently large.

**Proof**

1. We need to choose the set $\mathcal{A}$ suitably.
2. Fix $\epsilon > 0$ and take $\mathcal{A} = W^n_{[X]_\epsilon}$ and hence $M = |\mathcal{A}|$.
3. For sufficiently large $n$, by WAEP,

$$(1-\epsilon)2^n(H(X) - \epsilon) \leq M = |\mathcal{A}| = |W^n_{[X]_\epsilon}| \leq 2^n(H(X) + \epsilon).$$

4. The coding rate satisfies

$$\frac{1}{n} \log(1 - \epsilon) + H(X) - \epsilon \leq \frac{1}{n} \log M \leq H(X) + \epsilon.$$

5. By WAEP,

$$P_e = \Pr\{X \notin \mathcal{A}\} = \Pr\{X \notin W^n_{[X]_\epsilon}\} < \epsilon.$$

6. Letting $\epsilon \to 0$, the coding rate tends to $H(X)$, while $P_e$ tends to 0.

---

**Theorem 5.2 (Weak AEP II)**

1) If $x \in W^n_{[X]_\epsilon}$, then

$$2^{-n(H(X) + \epsilon)} \leq p(x) \leq 2^{-n(H(X) - \epsilon)}.$$

2) For $n$ sufficiently large,

$$\Pr\{\mathbf{X} \in W^n_{[X]_\epsilon}\} > 1 - \epsilon.$$

3) For $n$ sufficiently large,

$$(1-\epsilon)2^n(H(X) - \epsilon) \leq |W^n_{[X]_\epsilon}| \leq 2^n(H(X) + \epsilon).$$
Source Coding Theorem (Direct Part)
For arbitrarily small $P_e$, there exists a block code whose coding rate is arbitrarily close to $H(X)$ when $n$ is sufficiently large.

Proof
1. We need to choose the set $\mathcal{A}$ suitably.
2. Fix $\varepsilon > 0$ and take $\mathcal{A} = W^n_{[X]_\varepsilon}$ and hence $M = |\mathcal{A}|$.
3. For sufficiently large $n$, by WAEP,

$$(1-\varepsilon)2^n(H(X)-\varepsilon) \leq M = |\mathcal{A}| = |W^n_{[X]_\varepsilon}| \leq 2^n(H(X)+\varepsilon).$$

4. The coding rate satisfies

$$\frac{1}{n} \log(1-\varepsilon) + H(X) - \varepsilon \leq \frac{1}{n} \log M \leq H(X) + \varepsilon.$$

5. By WAEP,

$$P_e = \Pr\{X \not\in \mathcal{A}\} = \Pr\{X \not\in W^n_{[X]_\varepsilon}\} < \varepsilon.$$

6. Letting $\varepsilon \to 0$, the coding rate tends to $H(X)$, while $P_e$ tends to 0.
Source Coding Theorem (Direct Part)
For arbitrarily small $P_e$, there exists a block code whose coding rate is arbitrarily close to $H(X)$ when $n$ is sufficiently large.

**Proof**
1. We need to choose the set $\mathcal{A}$ suitably.
2. Fix $\epsilon > 0$ and take $\mathcal{A} = W^n_{[X]_{\epsilon}}$ and hence $M = |\mathcal{A}|$.
3. For sufficiently large $n$, by WAEP,
   \[ (1-\epsilon)2^n(H(X)-\epsilon) \leq M = |\mathcal{A}| = |W^n_{[X]_{\epsilon}}| \leq 2^n(H(X)+\epsilon). \]
4. The coding rate satisfies
   \[ \frac{1}{n} \log(1-\epsilon) + H(X) - \epsilon \leq \frac{1}{n} \log M \leq H(X) + \epsilon. \]
5. By WAEP,
   \[ P_e = \Pr\{X \notin \mathcal{A}\} = \Pr\{X \notin W^n_{[X]_{\epsilon}}\} < \epsilon. \]
6. Letting $\epsilon \to 0$, the coding rate tends to $H(X)$, while $P_e$ tends to 0.

**Theorem 5.2 (Weak AEP II)**
1) If $x \in W^n_{[X]_{\epsilon}}$, then
   \[ 2^{-n(H(X)+\epsilon)} \leq p(x) \leq 2^{-n(H(X)-\epsilon)}. \]
2) For $n$ sufficiently large,
   \[ \Pr\{X \in W^n_{[X]_{\epsilon}}\} > 1 - \epsilon. \]
3) For $n$ sufficiently large,
   \[ (1-\epsilon)2^n(H(X)-\epsilon) \leq |W^n_{[X]_{\epsilon}}| \leq 2^n(H(X)+\epsilon). \]
Converse

- We will prove the converse for the class of block codes we use for proving the direct part.
Converse

- We will prove the converse for the class of block codes we use for proving the direct part.

- For a general converse, see Problem 2.
Source Coding Theorem (Converse)
For any block code with block length $n$ and coding rate less than $H(X) - \zeta$, where $\zeta > 0$ does not change with $n$, then $P_e \to 1$ as $n \to \infty$.

Proof

Theorem 5.2 (Weak AEP II)

1) If $x \in W^n_{[X]_\epsilon}$, then

$$2^{-n(H(X)+\epsilon)} \leq p(x) \leq 2^{-n(H(X)-\epsilon)}.$$

2) For $n$ sufficiently large,

$$\Pr\{X \in W^n_{[X]_\epsilon}\} > 1 - \epsilon.$$

3) For $n$ sufficiently large,

$$(1 - \epsilon)2^n(H(X) - \epsilon) \leq |W^n_{[X]_\epsilon}| \leq 2^n(H(X) + \epsilon).$$
Source Coding Theorem (Converse)
For any block code with block length \( n \) and coding rate less than \( H(X) - \zeta \), where \( \zeta > 0 \) does not change with \( n \), then \( P_e \to 1 \) as \( n \to \infty \).

**Proof**

1. Consider any block code whose rate is less than \( H(X) - \zeta \), i.e.,
   \[
   H(X) + \varepsilon < \frac{1}{n} \log M < H(X) - \zeta,
   \]
   where \( \zeta > 0 \) does not change with \( n \). Then total number of codewords
   \[
   M < 2^n (H(X) - \zeta).
   \]

2. In general, some of the indices in \( I \) cover \( x \in W_n[X]_\varepsilon \), while the others cover \( x \notin W_n[X]_\varepsilon \).

3. By WAEP, the total probability of typical sequences covered is upper bounded by
   \[
   2^n (H(X) - \zeta) < 2^n (H(X) + \varepsilon).
   \]

4. By WAEP, the total probability covered by \( I \), i.e.,
   \[
   \Pr\{X \in W_n[X]_\varepsilon\},
   \]
   is upper bounded by
   \[
   2^n (H(X) - \zeta) + \Pr\{X \notin W_n[X]_\varepsilon\} < 2^n (H(X) + \varepsilon).
   \]

5. Then \( P_e = \Pr\{X \notin W_n[X]_\varepsilon\} > 1 \) holds for any \( \varepsilon > 0 \) and \( n \) sufficiently large.

6. Take \( \varepsilon < H(X) - \zeta \). Then \( P_e > 2^n \varepsilon \) for \( n \) sufficiently large.

7. Finally, let \( \varepsilon \to 0 \).

---

**Theorem 5.2 (Weak AEP II)**

1) If \( x \in W_n[X]_\varepsilon \), then
   \[
   2^{-n(H(X) + \varepsilon)} \leq p(x) \leq 2^{-n(H(X) - \varepsilon)}.
   \]

2) For \( n \) sufficiently large,
   \[
   \Pr\{X \in W_n[X]_\varepsilon\} > 1 - \varepsilon.
   \]

3) For \( n \) sufficiently large,
   \[
   (1 - \varepsilon)2^n(H(X) - \varepsilon) \leq |W_n[X]_\varepsilon| \leq 2^n(H(X) + \varepsilon).
   \]
Source Coding Theorem (Converse)
For any block code with block length $n$ and coding rate less than $H(X) - \zeta$, where $\zeta > 0$ does not change with $n$, then $P_e \to 1$ as $n \to \infty$.

Proof
1. Consider any block code whose rate is less than $H(X) - \zeta$, i.e.,

$$\frac{1}{n} \log M < H(X) - \zeta,$$

where $\zeta > 0$ does not change with $n$. Then total number of codewords

$$M \leq 2^{n(H(X) - \zeta)}.$$

2. In general, some of the indices in $I$ cover $x \in W^n_{[X]\epsilon}$, while the others cover $x \not\in W^n_{[X]\epsilon}$.

3. By WAEP, the total probability of typical sequences covered is upper bounded by

$$2^n(H(X) - \zeta) \leq 2^n(H(X) + \epsilon) + \Pr\{X \in W^n_{[X]\epsilon}\} < 2^n(H(X) - \zeta).$$

4. By WAEP, the total probability covered by $I$, i.e.,

$$\Pr\{X \in W^n_{[X]\epsilon}\} < 2^n(H(X) - \zeta) + \epsilon.$$

5. Then $P_e = \Pr\{X \not\in A\} > 1$ holds for any $\epsilon > 0$ and $n$ sufficiently large.

6. Take $\epsilon < H(X) - \zeta$. Then $P_e > 1$ for $n \geq (H(X) - \zeta)$ sufficiently large.

7. Finally, let $\epsilon \to 0$.

Theorem 5.2 (Weak AEP II)

1) If $x \in W^n_{[X]\epsilon}$, then

$$2^{-n(H(X) + \epsilon)} \leq p(x) \leq 2^{-n(H(X) - \epsilon)}.$$

2) For $n$ sufficiently large,

$$\Pr\{X \in W^n_{[X]\epsilon}\} > 1 - \epsilon.$$

3) For $n$ sufficiently large,

$$(1 - \epsilon)2^{n(H(X) - \epsilon)} \leq |W^n_{[X]\epsilon}| \leq 2^{n(H(X) + \epsilon)}.$$
Source Coding Theorem (Converse)
For any block code with block length \( n \) and coding rate less than \( H(X) - \zeta \), where \( \zeta > 0 \) does not change with \( n \), then \( P_e \to 1 \) as \( n \to \infty \).

**Proof**
1. Consider any block code whose rate is less than \( H(X) - \zeta \), i.e.,
   \[
   \frac{1}{n} \log M < H(X) - \zeta,
   \]
   where \( \zeta > 0 \) does not change with \( n \). Then the total number of codewords
   \[
   M \leq 2^{n(H(X) - \zeta)}.
   \]

Theorem 5.2 (Weak AEP II)

1) If \( x \in W^n_{[X]} \epsilon \), then
   \[
   2^{-n(H(X)+\epsilon)} \leq p(x) \leq 2^{-n(H(X)-\epsilon)}.
   \]

2) For \( n \) sufficiently large,
   \[
   \Pr\{X \in W^n_{[X]} \epsilon \} > 1 - \epsilon.
   \]

3) For \( n \) sufficiently large,
   \[
   (1 - \epsilon)2^{n(H(X) - \epsilon)} \leq |W^n_{[X]} \epsilon| \leq 2^{n(H(X) + \epsilon)}.
   \]
Source Coding Theorem (Converse)
For any block code with block length $n$ and coding rate less than $H(X) - \zeta$, where $\zeta > 0$ does not change with $n$, then $P_e \to 1$ as $n \to \infty$.

Proof
1. Consider any block code whose rate is less than $H(X) - \zeta$, i.e.,
\[
\frac{1}{n} \log M < H(X) - \zeta,
\]
where $\zeta > 0$ does not change with $n$. Then total number of codewords
\[
M \leq 2^{n(H(X) - \zeta)}.
\]

2. In general, some of the indices in $I$ cover $x \in W^n_{[X]_\epsilon}$, while the others cover $x \not\in W^n_{[X]_\epsilon}$.

Theorem 5.2 (Weak AEP II)
1) If $x \in W^n_{[X]_\epsilon}$, then
\[
2^{-n(H(X) + \epsilon)} \leq p(x) \leq 2^{-n(H(X) - \epsilon)}.
\]

2) For $n$ sufficiently large,
\[
\Pr\{X \in W^n_{[X]_\epsilon}\} > 1 - \epsilon.
\]

3) For $n$ sufficiently large,
\[
(1 - \epsilon)2^{n(H(X) - \epsilon)} \leq |W^n_{[X]_\epsilon}| \leq 2^{n(H(X) + \epsilon)}.
\]
Source Coding Theorem (Converse)
For any block code with block length $n$ and coding rate less than $H(X) - \zeta$, where $\zeta > 0$ does not change with $n$, then $P_e \to 1$ as $n \to \infty$.

Proof
1. Consider any block code whose rate is less than $H(X) - \zeta$, i.e.,

$$
\frac{1}{n} \log M < H(X) - \zeta,
$$

where $\zeta > 0$ does not change with $n$. Then total number of codewords

$$
M \leq 2^n(H(X) - \zeta).
$$

2. In general, some of the indices in $I$ cover $x \in W^n_{[X] \epsilon}$, while the others cover $x \not\in W^n_{[X] \epsilon}$.

Theorem 5.2 (Weak AEP II)
1) If $x \in W^n_{[X] \epsilon}$, then

$$
2^{-n(H(X) + \epsilon)} \leq p(x) \leq 2^{-n(H(X) - \epsilon)}.
$$

2) For $n$ sufficiently large,

$$
\Pr\{X \in W^n_{[X] \epsilon}\} > 1 - \epsilon.
$$

3) For $n$ sufficiently large,

$$
(1 - \epsilon)2^n(H(X) - \epsilon) \leq |W^n_{[X] \epsilon}| \leq 2^n(H(X) + \epsilon).
$$
**Source Coding Theorem (Converse)**

For any block code with block length $n$ and coding rate less than $H(X) - \zeta$, where $\zeta > 0$ does not change with $n$, then $P_e \to 1$ as $n \to \infty$.

**Proof**

1. Consider any block code whose rate is less than $H(X) - \zeta$, i.e.,

\[
\frac{1}{n} \log M < H(X) - \zeta,
\]

where $\zeta > 0$ does not change with $n$. Then total number of codewords

\[
M \leq 2^{n(H(X) - \zeta)}.
\]

2. In general, some of the indices in $\mathcal{I}$ cover $x \in W_{[X] \epsilon}^n$, while the others cover $x \not\in W_{[X] \epsilon}^n$.

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**Theorem 5.2 (Weak AEP II)**

1) If $x \in W_{[X] \epsilon}^n$, then

\[
2^{-n(H(X) + \epsilon)} \leq p(x) \leq 2^{-n(H(X) - \epsilon)}.
\]

2) For $n$ sufficiently large,

\[
\Pr\{x \in W_{[X] \epsilon}^n\} > 1 - \epsilon.
\]

3) For $n$ sufficiently large,

\[
(1 - \epsilon)2^{n(H(X) - \epsilon)} \leq |W_{[X] \epsilon}^n| \leq 2^{n(H(X) + \epsilon)}.
\]
Source Coding Theorem (Converse)
For any block code with block length $n$ and coding rate less than $H(X) - \zeta$, where $\zeta > 0$ does not change with $n$, then $P_e \rightarrow 1$ as $n \rightarrow \infty$.

Proof
1. Consider any block code whose rate is less than $H(X) - \zeta$, i.e.,

$$\frac{1}{n} \log M < H(X) - \zeta,$$

where $\zeta > 0$ does not change with $n$. Then total number of codewords

$$M \leq 2^{n(H(X) - \zeta)}.$$

2. In general, some of the indices in $I$ cover $x \in W^n_{[X]_\epsilon}$, while the others cover $x \not\in W^n_{[X]_\epsilon}$.

Theorem 5.2 (Weak AEP II)

1) If $x \in W^n_{[X]_\epsilon}$, then

$$2^{-n(H(X)+\epsilon)} \leq p(x) \leq 2^{-n(H(X)-\epsilon)}.$$

2) For $n$ sufficiently large,

$$\Pr\{ X \in W^n_{[X]_\epsilon} \} > 1 - \epsilon.$$

3) For $n$ sufficiently large,

$$(1 - \epsilon)2^{n(H(X) - \epsilon)} \leq |W^n_{[X]_\epsilon}| \leq 2^{n(H(X) + \epsilon)}.$$

\[ \chi^n \]

\[ A \]

\[ I \]

\[ W^n_{[X]_\epsilon} \]
**Source Coding Theorem (Converse)**

For any block code with block length $n$ and coding rate less than $H(X) - \zeta$, where $\zeta > 0$ does not change with $n$, then $P_e \to 1$ as $n \to \infty$.

**Proof**

1. Consider any block code whose rate is less than $H(X) - \zeta$, i.e.,

$$\frac{1}{n} \log M < H(X) - \zeta,$$

where $\zeta > 0$ does not change with $n$. Then total number of codewords

$$M \leq 2^{n(H(X) - \zeta)}.$$

2. In general, some of the indices in $I$ cover $x \in W^n[X]_\epsilon$, while the others cover $x \notin W^n[X]_\epsilon$.

3. By WAEP, the total probability of typical sequences covered is upper bounded by

$$2^n(H(X) - \zeta)2^{-n(H(X) - \epsilon)} = 2^{-n(\zeta - \epsilon)}.$$

---

**Theorem 5.2 (Weak AEP II)**

1. If $x \in W^n[X]_\epsilon$, then

$$2^{-n(H(X) + \epsilon)} \leq p(x) \leq 2^{-n(H(X) - \epsilon)}.$$

2. For $n$ sufficiently large,

$$\Pr\{X \in W^n[X]_\epsilon\} > 1 - \epsilon.$$

3. For $n$ sufficiently large,

$$(1 - \epsilon)2^n(H(X) - \epsilon) \leq |W^n[X]_\epsilon| \leq 2^n(H(X) + \epsilon).$$
Source Coding Theorem (Converse)
For any block code with block length \( n \) and coding rate less than \( H(X) - \zeta \), where \( \zeta > 0 \) does not change with \( n \), then \( P_e \rightarrow 1 \) as \( n \rightarrow \infty \).

Proof
1. Consider any block code whose rate is less than \( H(X) - \zeta \), i.e.,

\[
\frac{1}{n} \log M < H(X) - \zeta,
\]

where \( \zeta > 0 \) does not change with \( n \). Then total number of codewords

\[
M \leq 2^{n(H(X) - \zeta)}.
\]

2. In general, some of the indices in \( \mathcal{I} \) cover \( x \in W^n_{[X]\epsilon} \), while the others cover \( x \not\in W^n_{[X]\epsilon} \).

3. By WAEP, the total probability of typical sequences covered is upper bounded by

\[
2^n(H(X) - \zeta)2^{-n(H(X) - \epsilon)} = 2^{-n(\zeta - \epsilon)}.
\]

Theorem 5.2 (Weak AEP II)
1) If \( x \in W^n_{[X]\epsilon} \), then

\[
2^{-n(H(X) + \epsilon)} \leq p(x) \leq 2^{-n(H(X) - \epsilon)}.
\]

2) For \( n \) sufficiently large,

\[
\Pr\{X \in W^n_{[X]\epsilon}\} > 1 - \epsilon.
\]

3) For \( n \) sufficiently large,

\[
(1 - \epsilon)2^n(H(X) - \epsilon) \leq |W^n_{[X]\epsilon}| \leq 2^n(H(X) + \epsilon).
\]
Source Coding Theorem (Converse)
For any block code with block length \( n \) and coding rate less than \( H(X) - \zeta \), where \( \zeta > 0 \) does not change with \( n \), then \( P_e \to 1 \) as \( n \to \infty \).

Proof
1. Consider any block code whose rate is less than \( H(X) - \zeta \), i.e.,

\[
\frac{1}{n} \log M < H(X) - \zeta,
\]

where \( \zeta > 0 \) does not change with \( n \). Then total number of codewords

\[
M \leq 2^{n(H(X) - \zeta)}.
\]

2. In general, some of the indices in \( I \) cover \( x \in W_{[X]_\epsilon}^n \), while the others cover \( x \not\in W_{[X]_\epsilon}^n \).

3. By WAEP, the total probability of typical sequences covered is upper bounded by

\[
2^n(H(X) - \zeta)2^{-n(H(X) - \epsilon)} = 2^{-n(\zeta - \epsilon)}.
\]

Theorem 5.2 (Weak AEP II)

1) If \( x \in W_{[X]_\epsilon}^n \), then

\[
2^{-n(H(X) + \epsilon)} \leq p(x) \leq 2^{-n(H(X) - \epsilon)}.
\]

2) For \( n \) sufficiently large,

\[
\Pr\{x \in W_{[X]_\epsilon}^n\} > 1 - \epsilon.
\]

3) For \( n \) sufficiently large,

\[
(1 - \epsilon)2^{n(H(X) - \epsilon)} \leq |W_{[X]_\epsilon}^n| \leq 2^{n(H(X) + \epsilon)}.
\]
Source Coding Theorem (Converse)

For any block code with block length \( n \) and coding rate less than \( H(X) - \zeta \), where \( \zeta > 0 \) does not change with \( n \), then \( P_e \to 1 \) as \( n \to \infty \).

Proof

1. Consider any block code whose rate is less than \( H(X) - \zeta \), i.e.,

\[
\frac{1}{n} \log M < H(X) - \zeta,
\]

where \( \zeta > 0 \) does not change with \( n \). Then total number of codewords

\[
M \leq 2^{n(H(X) - \zeta)}.
\]

2. In general, some of the indices in \( I \) cover \( x \in W^n_{[X]_\epsilon} \), while the others cover \( x \not\in W^n_{[X]_\epsilon} \).

3. By WAEP, the total probability of typical sequences covered is upper bounded by

\[
2^{n(H(X) - \zeta)} 2^{-n(H(X) - \epsilon)} = 2^{-n(\zeta - \epsilon)}.
\]

Theorem 5.2 (Weak AEP II)

1) If \( x \in W^n_{[X]_\epsilon} \), then

\[
2^{-n(H(X) + \epsilon)} \leq p(x) \leq 2^{-n(H(X) - \epsilon)}.
\]

2) For \( n \) sufficiently large,

\[
\Pr\{X \in W^n_{[X]_\epsilon}\} > 1 - \epsilon.
\]

3) For \( n \) sufficiently large,

\[
(1 - \epsilon)2^{n(H(X) - \epsilon)} \leq |W^n_{[X]_\epsilon}| \leq 2^{n(H(X) + \epsilon)}.
\]
Source Coding Theorem (Converse)
For any block code with block length \(n\) and coding rate less than \(H(X) - \zeta\), where \(\zeta > 0\) does not change with \(n\), then \(P_e \to 1\) as \(n \to \infty\).

Proof
1. Consider any block code whose rate is less than \(H(X) - \zeta\), i.e.,
\[
\frac{1}{n} \log M < H(X) - \zeta,
\]
where \(\zeta > 0\) does not change with \(n\). Then total number of codewords
\[
M \leq 2^n(H(X) - \zeta).
\]
2. In general, some of the indices in \(I\) cover \(x \in W^n_{[X]\epsilon}\), while the others cover \(x \not\in W^n_{[X]\epsilon}\).
3. By WAEP, the total probability of typical sequences covered is upper bounded by
\[
2^n(H(X) - \zeta)2^{-n(H(X) - \epsilon)} = 2^{-n(\zeta - \epsilon)}.
\]

Theorem 5.2 (Weak AEP II)
1) If \(x \in W^n_{[X]\epsilon}\), then
\[
2^{-n(H(X)+\epsilon)} \leq p(x) \leq 2^{-n(H(X)-\epsilon)}.
\]
2) For \(n\) sufficiently large,
\[
\Pr\{x \in W^n_{[X]\epsilon}\} > 1 - \epsilon.
\]
3) For \(n\) sufficiently large,
\[
(1 - \epsilon)2^n(H(X) - \epsilon) \leq |W^n_{[X]\epsilon}| \leq 2^n(H(X) + \epsilon).
\]
Source Coding Theorem (Converse)
For any block code with block length $n$ and coding rate less than $H(X) - \zeta$, where $\zeta > 0$ does not change with $n$, then $P_e \to 1$ as $n \to \infty$.

**Proof**
1. Consider any block code whose rate is less than $H(X) - \zeta$, i.e.,
$$ \frac{1}{n} \log M < H(X) - \zeta, $$
where $\zeta > 0$ does not change with $n$. Then total number of codewords
$$ M \leq 2^{n(H(X)-\zeta)}. $$

2. In general, some of the indices in $I$ cover $x \in W_n^{[X]_\epsilon}$, while the others cover $x \not\in W_n^{[X]_\epsilon}$.

3. By WAEP, the total probability of typical sequences covered is upper bounded by
$$ \frac{2^n(H(X)-\zeta)}{2^{-n(H(X)-\epsilon)}} = 2^{-n(\zeta-\epsilon)}. $$

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**Theorem 5.2 (Weak AEP II)**

1) If $x \in W_n^{[X]_\epsilon}$, then
$$ 2^{-n(H(X)+\epsilon)} \leq p(x) \leq 2^{-n(H(X)-\epsilon)}. $$

2) For $n$ sufficiently large,
$$ \Pr\{X \in W_n^{[X]_\epsilon}\} > 1 - \epsilon. $$

3) For $n$ sufficiently large,
$$ (1 - \epsilon)2^{n(H(X)-\epsilon)} \leq |W_n^{[X]_\epsilon}| \leq 2^{n(H(X)+\epsilon)}. $$

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![Diagram](image-url)
Source Coding Theorem (Converse)
For any block code with block length \( n \) and coding rate less than \( H(X) - \zeta \), where \( \zeta > 0 \) does not change with \( n \), then \( P_e \to 1 \) as \( n \to \infty \).

Proof
1. Consider any block code whose rate is less than \( H(X) - \zeta \), i.e.,

\[
\frac{1}{n} \log M < H(X) - \zeta,
\]

where \( \zeta > 0 \) does not change with \( n \). Then total number of codewords

\[
M \leq 2^{n(H(X) - \zeta)}.
\]

2. In general, some of the indices in \( I \) cover \( x \in W^n_{[X]} \), while the others cover \( x \notin W^n_{[X]} \).

3. By WAEP, the total probability of typical sequences covered is upper bounded by

\[
2^{n(H(X) - \zeta)} 2^{-n(H(X) - \epsilon)} = 2^{-n(\zeta - \epsilon)}.
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Theorem 5.2 (Weak AEP II)
1) If \( x \in W^n_{[X]} \), then

\[
2^{-n(H(X) + \epsilon)} \leq p(x) \leq 2^{-n(H(X) - \epsilon)}.
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2) For \( n \) sufficiently large,

\[
Pr\{X \in W^n_{[X]}\} > 1 - \epsilon.
\]

3) For \( n \) sufficiently large,

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(1 - \epsilon)2^{n(H(X) - \epsilon)} \leq |W^n_{[X]}| \leq 2^{n(H(X) + \epsilon)}.
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1. Consider any block code whose rate is less than $H(X) - \zeta$, i.e.,
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1) If $x \in W^n_{[X]\epsilon}$, then
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3) For \( n \) sufficiently large,
\[
(1 - \epsilon)2^n(H(X) - \epsilon) \leq |W^n_{[X]_{\epsilon}}| \leq 2^n(H(X) + \epsilon).
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For any block code with block length \( n \) and coding rate less than \( H(X) - \zeta \), where \( \zeta > 0 \) does not change with \( n \), then \( P_e \to 1 \) as \( n \to \infty \).

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\[
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For any block code with block length \( n \) and coding rate less than \( H(X) - \zeta \), where \( \zeta > 0 \) does not change with \( n \), then \( P_e \to 1 \) as \( n \to \infty \).

**Proof**

1. Consider any block code whose rate is less than \( H(X) - \zeta \), i.e.,

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\[
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2. In general, some of the indices in \( \mathcal{I} \) cover \( x \in W^n_{[X]_{\epsilon}} \), while the others cover \( x \not\in W^n_{[X]_{\epsilon}} \).

3. By WAEP, the total probability of typical sequences covered is upper bounded by

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2^{n(H(X) - \zeta)}2^{-n(H(X) - \epsilon)} = 2^{-n(\zeta - \epsilon)}.
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4. By WAEP, the total probability covered by \( \mathcal{I} \), i.e., \( \Pr\{X \in \mathcal{A}\} \), is upper bounded by

\[
2^{-n(\zeta - \epsilon)} + \Pr\{X \not\in W^n_{[X]_{\epsilon}}\} < 2^{-n(\zeta - \epsilon)} + \epsilon.
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---

**Theorem 5.2 (Weak AEP II)**

1) If \( x \in W^n_{[X]_{\epsilon}} \), then

\[
2^{-n(H(X) + \epsilon)} \leq p(x) \leq 2^{-n(H(X) - \epsilon)}.
\]

2) For \( n \) sufficiently large,

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\Pr\{X \in W^n_{[X]_{\epsilon}}\} > 1 - \epsilon.
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For any block code with block length $n$ and coding rate less than $H(X) - \zeta$, where $\zeta > 0$ does not change with $n$, then $P_e \to 1$ as $n \to \infty$.

**Proof**
1. Consider any block code whose rate is less than $H(X) - \zeta$, i.e.,
\[
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M \leq 2^{n(H(X) - \zeta)}.
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2. In general, some of the indices in $\mathcal{I}$ cover $x \in W^n_{[X]\epsilon}$, while the others cover $x \not\in W^n_{[X]\epsilon}$.

3. By WAEP, the total probability of typical sequences covered is upper bounded by
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4. By WAEP, the total probability covered by $\mathcal{I}$, i.e., $Pr\{X \in A\}$, is upper bounded by
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1) If $x \in W^n_{[X]\epsilon}$, then
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2^{-n(H(X) + \epsilon)} \leq p(x) \leq 2^{-n(H(X) - \epsilon)}.
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For any block code with block length \( n \) and coding rate less than \( H(X) - \zeta \), where \( \zeta > 0 \) does not change with \( n \), then \( P_e \to 1 \) as \( n \to \infty \).

**Proof**
1. Consider any block code whose rate is less than \( H(X) - \zeta \), i.e.,
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2. In general, some of the indices in \( I \) cover \( x \in W^n_{[X]_\epsilon} \), while the others cover \( x \not\in W^n_{[X]_\epsilon} \).

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**Theorem 5.2 (Weak AEP II)**
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For any block code with block length $n$ and coding rate less than $H(X) - \zeta$, where $\zeta > 0$ does not change with $n$, then $P_e \to 1$ as $n \to \infty$.

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1. Consider any block code whose rate is less than $H(X) - \zeta$, i.e.,

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2. In general, some of the indices in $\mathcal{I}$ cover $x \in W^n_{[X] \epsilon}$, while the others cover $x \not\in W^n_{[X] \epsilon}$.

3. By WAEP, the total probability of typical sequences covered is upper bounded by

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1) If $x \in W^n_{[X] \epsilon}$, then

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2) For $n$ sufficiently large,

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For any block code with block length $n$ and coding rate less than $H(X) - \zeta$, where $\zeta > 0$ does not change with $n$, then $P_e \to 1$ as $n \to \infty$.

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1. Consider any block code whose rate is less than $H(X) - \zeta$, i.e.,
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2. In general, some of the indices in $I$ cover $x \in W^n_{[X] \epsilon}$, while the others cover $x \notin W^n_{[X] \epsilon}$.

3. By WAEP, the total probability of typical sequences covered is upper bounded by
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\]

4. By WAEP, the total probability covered by $I$, i.e.,
\[
\Pr\{X \in A\},
\]
is upper bounded by
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2. In general, some of the indices in $I$ cover $x \in W^n_{[X]\epsilon}$, while the others cover $x \not\in W^n_{[X]\epsilon}$.

3. By WAEP, the total probability of typical sequences covered is upper bounded by

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3. For $n$ sufficiently large,

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2. In general, some of the indices in $I$ cover $x \in W^n_{[X] \epsilon}$, while the others cover $x \not\in W^n_{[X] \epsilon}$.

3. By WAEP, the total probability of typical sequences covered is upper bounded by

$$2^{n(H(X) - \zeta)} 2^{-n(H(X) - \epsilon)} = 2^{-n(\zeta - \epsilon)}.$$

4. By WAEP, the total probability covered by $I$, i.e.,

$$\Pr\{X \in A\},$$

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$$2^{-n(\zeta - \epsilon)} + \Pr\{X \not\in W^n_{[X] \epsilon}\} < 2^{-n(\zeta - \epsilon)} + \epsilon.$$

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**Proof**

1. Consider any block code whose rate is less than $H(X) - \zeta$, i.e.,

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where $\zeta > 0$ does not change with $n$. Then total number of codewords

$$M \leq 2^{n(H(X)-\zeta)}.$$

2. In general, some of the indices in $I$ cover $x \in W^n_{[X]\epsilon}$, while the others cover $x \not\in W^n_{[X]\epsilon}$.

3. By WAEP, the total probability of typical sequences covered is upper bounded by

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1. Consider any block code whose rate is less than $H(X) - \zeta$, i.e.,

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2. In general, some of the indices in $I$ cover $x \in W^n_{[X]\epsilon}$, while the others cover $x \not\in W^n_{[X]\epsilon}$.

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4. By WAEP, the total probability covered by $I$, i.e., $\Pr\{X \in A\}$, is upper bounded by

$$2^{-n(\zeta - \epsilon)} + \Pr\{X \not\in W^n_{[X]\epsilon}\} < 2^{-n(\zeta - \epsilon)} + \epsilon.$$

5. Then $P_e = \Pr\{X \not\in A\} > 1 - (2^{-n(\zeta - \epsilon)} + \epsilon)$ holds for any $\epsilon > 0$ and $n$ sufficiently large.

---

**Theorem 5.2 (Weak AEP II)**

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For any block code with block length $n$ and coding rate less than $H(X) - \zeta$, where $\zeta > 0$ does not change with $n$, then $P_e \to 1$ as $n \to \infty$.

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1. Consider any block code whose rate is less than $H(X) - \zeta$, i.e.,
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   \frac{1}{n} \log M < H(X) - \zeta,
   \]
   where $\zeta > 0$ does not change with $n$. Then total number of codewords
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   M \leq 2^{n(H(X) - \zeta)}.
   \]

2. In general, some of the indices in $I$ cover $x \in W^n_{[X]e}$, while the others cover $x \not\in W^n_{[X]e}$.

3. By WAEP, the total probability of typical sequences covered is upper bounded by
   \[
   2^{n(H(X) - \zeta)} 2^{-n(H(X) - \epsilon)} = 2^{-n(\zeta - \epsilon)}.
   \]

4. By WAEP, the total probability covered by $I$, i.e., $\Pr\{X \in A\}$, is upper bounded by
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   2^{-n(\zeta - \epsilon)} + \Pr\{X \not\in W^n_{[X]e}\} < 2^{-n(\zeta - \epsilon)} + \epsilon.
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   \Pr\{X \in W^n_{[X]e}\} > 1 - \epsilon.
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**Proof**
1. Consider any block code whose rate is less than \( H(X) - \zeta \), i.e.,
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2. In general, some of the indices in \( I \) cover \( x \in W^n_{[X]_{\epsilon}} \), while the others cover \( x \not\in W^n_{[X]_{\epsilon}} \).

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\[
2^{-n(H(X) + \epsilon)} \leq p(x) \leq 2^{-n(H(X) - \epsilon)}.
\]

2) For \( n \) sufficiently large,
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Source Coding Theorem (Converse)
For any block code with block length $n$ and coding rate less than $H(X) - \zeta$, where $\zeta > 0$ does not change with $n$, then $P_e \rightarrow 1$ as $n \rightarrow \infty$.

Proof
1. Consider any block code whose rate is less than $H(X) - \zeta$, i.e.,
   \[
   \frac{1}{n} \log M < H(X) - \zeta,
   \]
   where $\zeta > 0$ does not change with $n$. Then total number of codewords
   \[
   M \leq 2^{n(H(X)-\zeta)}.
   \]
2. In general, some of the indices in $\mathcal{I}$ cover $x \in W^n_{[X][\epsilon]}$, while the others cover $x \not\in W^n_{[X][\epsilon]}$.
3. By WAEP, the total probability of typical sequences covered is upper bounded by
   \[
   2^{n(H(X)-\zeta)} 2^{-n(H(X)-\epsilon)} = 2^{-n(\zeta-\epsilon)}.
   \]
4. By WAEP, the total probability covered by $\mathcal{I}$, i.e., $\Pr\{X \in \mathcal{A}\}$, is upper bounded by
   \[
   2^{-n(\zeta-\epsilon)} + \Pr\{X \not\in W^n_{[X][\epsilon]}\} < 2^{-n(\zeta-\epsilon)} + \epsilon.
   \]
5. Then $P_e = \Pr\{X \not\in \mathcal{A}\} > 1 - (2^{-n(\zeta-\epsilon)} + \epsilon)$ holds for any $\epsilon > 0$ and $n$ sufficiently large.
6. Take $\epsilon < \zeta$. Then $P_e > 1 - 2\epsilon$ for $n(\epsilon)$ sufficiently large.

Theorem 5.2 (Weak AEP II)
1) If $x \in W^n_{[X][\epsilon]}$, then
   \[
   2^{-n(H(X)+\epsilon)} \leq p(x) \leq 2^{-n(H(X)-\epsilon)}.
   \]
2) For $n$ sufficiently large,
   \[
   \Pr\{X \in W^n_{[X][\epsilon]}\} > 1 - \epsilon.
   \]
3) For $n$ sufficiently large,
   \[
   (1-\epsilon)2^{n(H(X)-\epsilon)} \leq |W^n_{[X][\epsilon]}| \leq 2^{n(H(X)+\epsilon)}.
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